LEARNING TO USE ARITHMETIC

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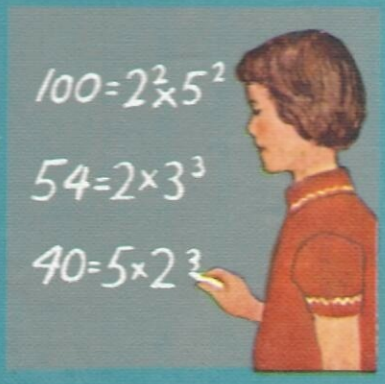

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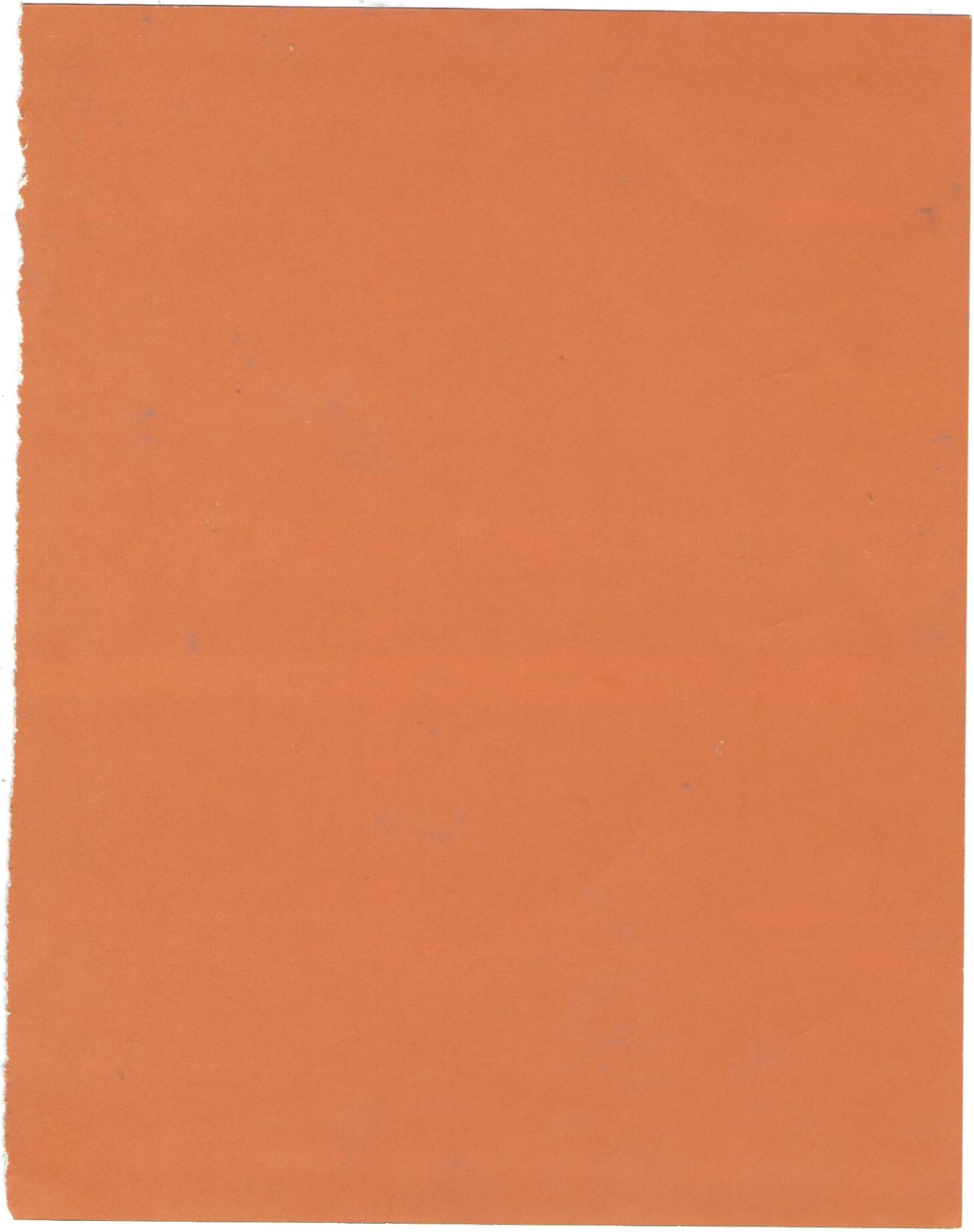

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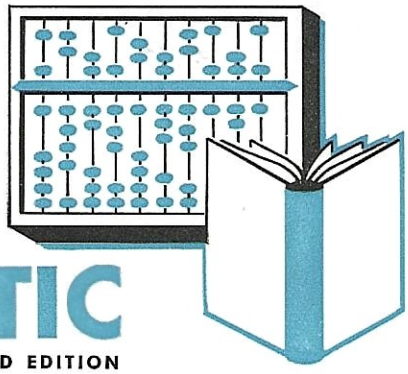
REVISED EDITION



BEGINNERS BOOK

BOOK 1

BOOK 2

BOOK 3

BOOK 4

BOOK 5

BOOK 6

BOOK 7

BOOK 8

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LEARNING TO USE

BOOK 6

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ARITHMETIC

REVISED EDITION

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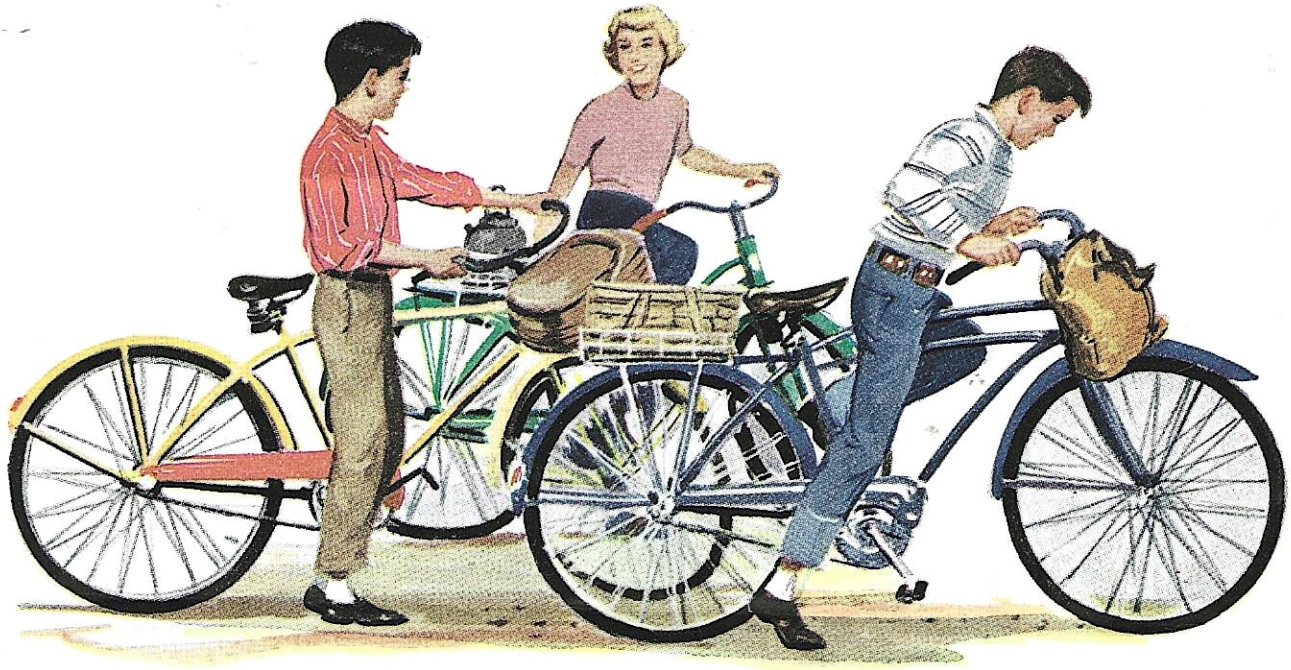

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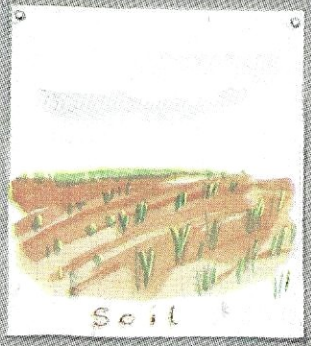
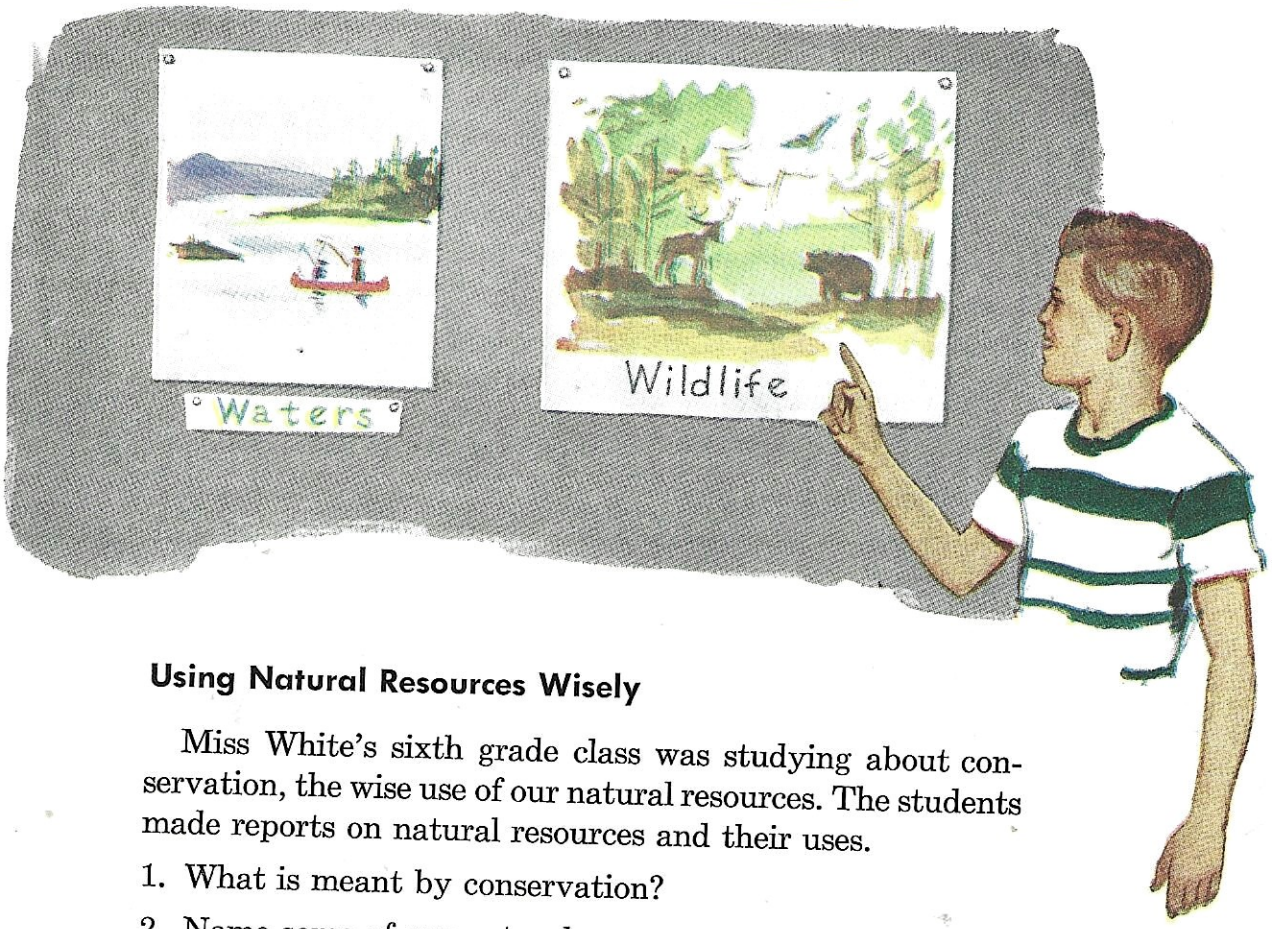


The Conservation Pledge

I give my pledge as an American to defend from waste, to work for wise use and good management of my country's natural resources – its soil and minerals, its forests, waters, and wildlife.

Wise Use of Natural Resources.

Using Large Numbers



Using Natural Resources Wisely

Miss White's sixth grade class was studying about conservation, the wise use of our natural resources. The students made reports on natural resources and their uses.

1. What is meant by conservation?
2. Name some of our natural resources.
3. Tell some uses of water, soil, wildlife, minerals, forests, grasslands.

There are more than 165 million people in the United States. Since our natural resources produce most of the raw materials used in making all the things these people need, large numbers are used in talking about conservation and in working problems about it. In this chapter, you will work with some of the large numbers that Miss White's students used in their reports.

Land Used for Crops

READING AND WRITING WHOLE NUMBERS TO 9 PLACES

In her report Carol told the class that the chief natural resource of the United States was the land on which crops grow. She also said that millions of acres of land have been damaged by soil erosion and careless use. Today only about 460,000,000 acres of first-class cropland remain. Every year the rivers wash away cropland that could be used for 125,000 160-acre farms. She ended her report by asking the question: If land continues to be carelessly used, will there always be enough crops for the people in the United States?

Can you read the numbers given in the statement above?

1. To read 125,000, say _?_ thousand. How many places does this number have?
2. To read 460,000,000, say _?_ million. How many places does this number have? Do you say the name of each place? Why not?
3. To read a number like 27030894, begin at the right and use commas to separate the number into groups of three figures, like this: 27,030,894. Now read the number. Say _?_ million, _?_ thousand, _?_. What did you say for the 0 in hundred thousands place?

Copy the following numbers. Place commas correctly and read the numbers.

4.	58645	32178825	133169642
5.	267931	4920000	703652003

Write these numbers in figures. Take care with the zeros.

6.	38 million	90 million	6 million
7.	257 million	3 million	74 million
8.	480 million	100 million	509 million

9. 14 million 200 thousand 700 million 401 thousand
10. 2 million 8 thousand 400 million 10 thousand 7
11. Five hundred thirty million, ninety-six thousand
12. Two hundred eighty thousand, nine hundred two
13. Seventeen million, five hundred sixty
14. Eight hundred million
15. Sixty-six thousand, thirteen
16. Three hundred forty million, nine hundred thousand, two hundred twenty

There is another way to write large numbers, a way that saves space and makes the number easy to read and to use. Newspapers often use this way of writing numbers.

17. Start with 1,000,000 and by dividing find $\frac{1}{2}$ million, $\frac{1}{4}$ million. \$3$\frac{1}{2}$ million is read three and one-half million dollars and equals \$3,500,000.
18. Write in figures: \$1$\frac{1}{2}$ million, \$2$\frac{1}{4}$ million, \$6$\frac{1}{2}$ million. Which numbers are easier to compare, your answers or the original numbers in the problem?
19. By changing the decimals to common fractions having the same value, find .5 million, .25 million.
20. Write in figures: \$1.5 million, \$2.25 million, \$6.5 million.

Large numbers are often used in rounded form. With numbers in the millions, several hundreds or even several thousands do not have much effect on the size of the number. It is wasted effort to use numbers that are more exact than we really need for our purpose, and it is often impossible to find the exact number for the information we want. For example, it would be impossible to find the exact population of the United States. Can you tell why this would be so? Also it is not possible to measure the exact distance from the earth to the sun. Is all measurement approximate? Why?

Newsprint from the Forest

ONE-PLACE NUMBERS TO 6 ADDENDS

John thought that he would like to learn more about the products used for newspapers. These are some of the facts that he learned. The paper that daily newspapers are printed on is called newsprint. Newsprint is made from the wood pulp of certain trees, such as the red spruce. On six days, from Monday through Saturday, the number of acres of forest used to make paper for printing a daily newspaper in a large city were as follows: 7, 6, 8, 9, 6, 5. What was the total number of acres of trees cut down to make the paper needed for this one week?

7
6
8
9
6
5
41

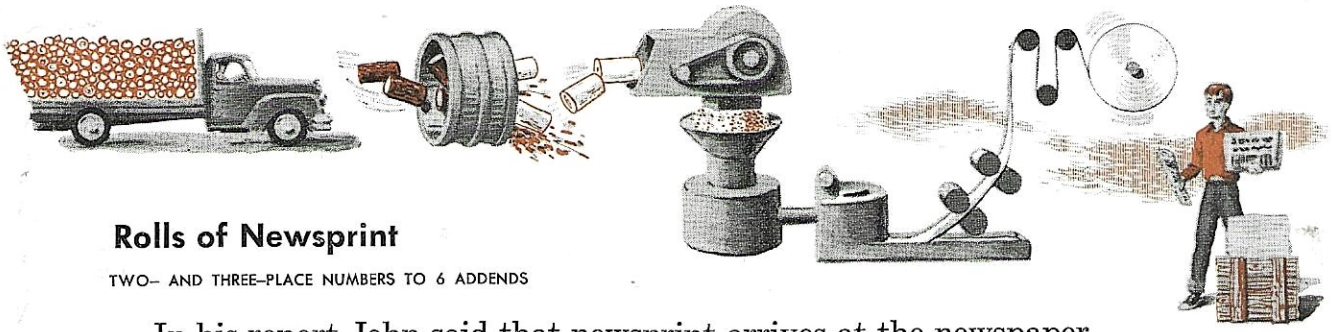
To find the answer, add. Why?

Add down. Think: 7, 13, 21, 30, 36, 41. Write 41.

To check your answer, add up. Think: 5, 11, 20, 28, 34, 41. If the work is correct, the sum will be the same as when you added down. 41 acres of forest were cut down to make the paper needed for this one week.

Write the sums for these examples on folded paper. Add down. Check by adding up.

1.	9	5	5	6	8	7
	7	0	7	5	6	8
	6	8	4	7	4	7
	5	1	8	9	5	9
				1	0	0
2.	4	0	7	8	6	9
	3	4	5	6	9	2
	9	0	3	4	7	5
	6	5	2	9	4	9
	2	8	8	0	5	8
	4	6	8	7	1	7



Rolls of Newsprint

TWO- AND THREE-PLACE NUMBERS TO 6 ADDENDS

In his report John said that newsprint arrives at the newspaper printing shop in rolls. From Monday through Saturday of one week a newspaper office needed the following numbers of rolls of newsprint: Monday, 226; Tuesday, 192; Wednesday, 256; Thursday, 288; Friday, 192; and Saturday, 160. How many rolls of newsprint were needed during that week?

To find the answer, add. Why?

First add the ones. Add down. Think: 6, 8, 14, 22, 24. Write 4 in the ones place. Keep the 2 tens to add with the other tens.

Next add the tens. Think: 2, 4, 13, 18, 26, 35, 41. Write 1 in the tens place. Keep the 4 hundreds to add with the other hundreds.

Then add the hundreds. Think: 4, 6, 7, 9, 11, 12, 13. Write 13.

There were 1314 rolls of newsprint needed for that week.

```
 226
 192
 256
 288
 192
 160
1314
```

Copy the examples below. Be careful to copy ones under ones, tens under tens, and so forth. Add down. Check by adding up.

1.	51	43	57	84	45	246
	35	25	64	26	67	811
	22	50	14	90	89	574
	48	38	48	57	12	343
	16	17	30	11	24	455
	40	51	29	38	56	318
2.	184	492	503	713	640	381
	379	723	167	564	358	57
	650	645	354	208	93	514
	542	166	289	451	46	90
	125	300	631	700	500	738
	437	578	342	396	467	66

Mining Coal

FOUR–PLACE NUMBERS TO 5 ADDENDS

Jane chose to give a report about coal. These are some of the facts she gave. About half of the known coal beds in the world are found in the United States. Coal mines can be seen in many parts of the nation.

Jane found that reference books often list figures in terms of so many 1000 tons, $1 million, 1000 barrels, and the like. For example, she found that in a five-year period, the state of Wyoming produced the following amounts, in 1000 tons, of coal annually: 5808; 6646; 8133; 9155; 9665. She wanted to find the total amount of coal produced in Wyoming in those five years.

```
 5808
 6646
 8133
 9155
 9665
39407
```

To find the answer, Jane added. Her example is shown at the left. Check to see that she added correctly. Add the figures in each place beginning at the right.

The answer is 39,407 in 1000 tons. Jane says that this means 39,407 thousands of tons, or 39,407,000 tons.

Copy the examples below. Be careful to copy ones under ones, tens under tens, and so forth. Add down. Check by adding up.

1.	3821	7375	5678	6948	4503
	2972	4753	2907	1354	1928
	5564	3510	4762	7031	6108
	6781	8483	5767	4785	5044
				6782	8753
2.	2584	1749	8195	4026	7864
	8670	5308	237	516	476
	9242	4062	9463	48	90
	3005	8503	874	7450	4559
	8983	7450	3905	5936	382

3. How would you write your answer to the first example in row 1 if each number means 1000 bushels? How would you write your answer to the first example in row 2 if each number means 1000 acres?

Cities Use Water

FIVE-PLACE NUMBERS TO 4 ADDENDS

Since water is one of our most important natural resources, Joe decided to find out about the use of water in his town. He visited the local water company to see how much water the people in his town used. The records of the company showed that water was used as follows: 70,242 gallons on Wednesday; 75,497 gallons on Thursday; 78,302 gallons on Friday; and 62,450 gallons on Saturday. What was the total amount of water used in these four days?

To find the answer, add down. Check your work by adding up. During the four days 286,491 gallons of water were used.

70,242
75,497
78,302
62,450
286,491

Copy, add, and check.

1.	59,394	75,196	30,856	86,589
	82,271	43,007	12,743	30,062
	25,086	68,781	93,600	74,170
2.	83,413	21,302	77,214	34,969
	15,058	47,513	59,632	80,548
	42,694	58,296	11,706	15,080
	67,382	94,001	85,451	68,787
3.	14,818	93,505	47,371	5,368
	35,290	584	930	513
	5,627	42,139	82,358	947
	9,423	8,242	787	12,089

4. In your house, do you use about the same amount of water every day? If not, on which days do you use more water? Why?

5. Joe found out that businesses and industries use much more water than all private homes put together. Look up paper making, sulfur mining, the manufacture of coke, or some other business or manufacturing process and find out how they use water. See if the amount of water they use surprises you. If possible, compare the amount they use in an hour with the amount your family uses in a month.

Our Forests

SIX-PLACE NUMBERS TO 3 ADDENDS

While doing his report on wood pulp used for newspapers, John learned that about one third of the land in the United States is covered by forests. He read that almost every state has large areas of forest land. For example, he found that in Illinois there are 812,654 acres, in Indiana 781,467 acres, and in Iowa 218,671 acres of forest land. What is the total forest area in these three states?

812,654
781,467
218,671
1,812,792

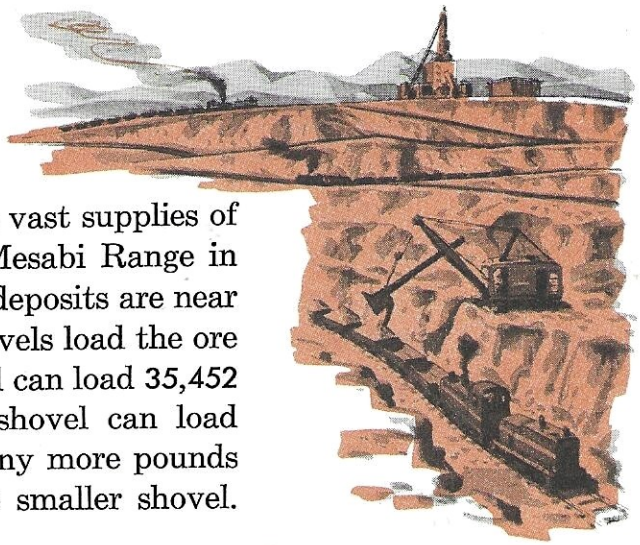
To find the answer, add down. Check your work by adding up. There are 1,812,792 acres of forest lands in the three states.

Copy the examples below, add, and check.

1.	6,405	13,817	13,975	52,952
	2,793	54,026	60,701	98,706
	8,614	83,195	74,312	40,063
2.	531,614	207,209	948,610	436,538
	753,000	878,032	432,109	708,136
	926,359	344,269	157,058	510,493
3.	112,793	687,549	208,345	756,953
	64,507	56,407	963,290	157,431
	831,976	32,428	470,016	307,128

Mining Iron Ore

FIVE- AND SIX-PLACE MINUENDS



Anne learned from her reading that vast supplies of the world's iron ore come from the Mesabi Range in Minnesota. She also read that the ore deposits are near the surface of the earth. Big steam shovels load the ore into waiting freight cars. A large shovel can load 35,452 pounds at one time, and a smaller shovel can load 27,658 pounds. She wondered how many more pounds the larger shovel could load than the smaller shovel.

To find the answer, subtract.

First subtract the ones. You cannot subtract 8 ones from 2 ones. Take 1 of the 5 tens, leaving 4 tens. This is often called borrowing. Change the 1 ten to 10 ones. 10 ones and 2 ones = 12 ones. 12 − 8 = 4. Write 4 in the ones place.

35,452 minuend
27,658 subtrahend
7,794 difference
35,452 check

Then subtract the tens. You cannot subtract 5 tens from 4 tens. Take 1 of the 4 hundreds and change it to 10 tens. 10 tens and 4 tens = 14 tens. 14 − 5 = 9. Write 9 in the tens place.

In the same way subtract the hundreds, the thousands, and the ten thousands. Check by adding the subtrahend and the difference. Their sum should equal the minuend.

Copy, subtract, and check.

1.	28,384	56,420	62,873	85,000	47,812
	17,596	40,675	34,097	66,579	27,906
2.	13,737	94,014	10,682	33,007	75,631
	6,968	74,537	3,957	22,468	4,687
3.	586,175	400,763	606,354	394,651	274,300
	138,368	205,806	557,479	366,786	163,587
4.	328,944	860,046	270,563	463,050	135,702
	27,998	604,707	70,674	403,387	90,666

Automobiles

SEVEN-PLACE MINUENDS

Bill read that the automobile industry uses huge amounts of raw materials. About one fifth of all the nation's steel, four fifths of its rubber, three fourths of its plate glass, one third of its lead, and one seventh of its copper go into our cars. The industry also uses hides, lumber, wool, sugar cane, corn products, cotton and synthetic fibers made from cotton, and various plastics.

Bill was surprised to learn that each year the automobile industry uses, among other things, enough steel to build 175 skyscrapers like the Empire State Building and enough cotton to make 20 dresses for every girl and woman in the United States.

In a recent year, 7,323,214 motor vehicles were manufactured. Of these, 6,116,948 were passenger cars. The rest were trucks and buses. Bill wondered how many trucks and buses were manufactured that year.

```
 7,323,214
 6,116,948
 1,206,266
```

To find the answer, Bill subtracted. His example is shown at the left. Check it mentally to see whether he subtracted correctly.

1,206,266 trucks and buses were manufactured that year.

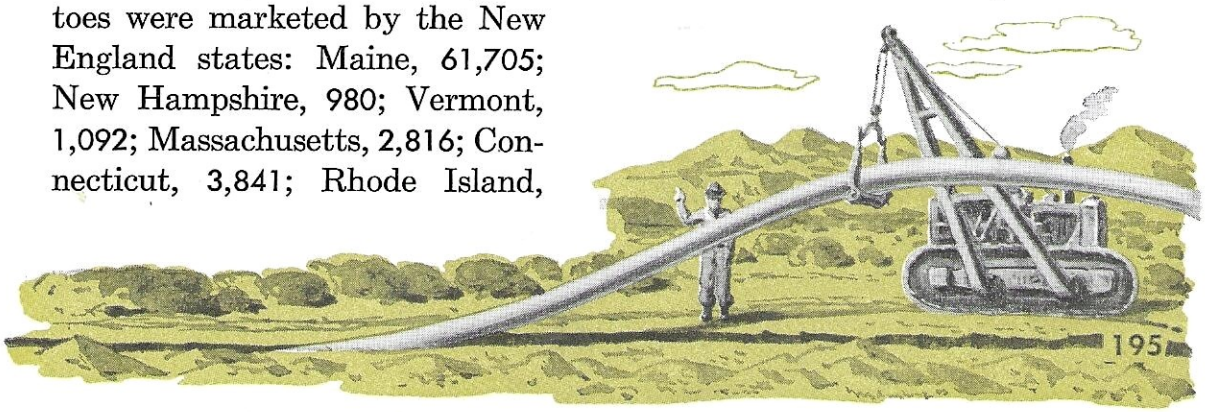
Copy, subtract, and check.

1.	222,862 125,875	738,305 507,546	3,558,178 2,148,699	6,103,703 4,091,857
2.	7,365,800 3,409,543	3,049,550 1,081,786	9,420,631 1,027,587	5,761,463 3,794,675
3.	8,457,815 368,917	1,062,100 835,621	4,708,520 850,763	7,600,481 764,817
4.	2,927,538 68,609	5,039,445 49,078	6,526,234 839,670	8,063,515 7,074,608

5. Explain why workers in many industries worry when automobile workers are laid off.

Problems in Addition and Subtraction

1. In a recent year Alabama produced 7,500,000 tons of iron ore. Wisconsin produced 1,700,000 tons of the ore. How many more tons did Alabama produce than Wisconsin?
2. How much lead was used in the United States during a five-year period if the yearly amounts were: 293,309 tons; 381,109 tons; 339,413 tons; 404,449 tons; and 433,000 tons?
3. In 1940 the production of bituminous coal in 1,000 tons was 394,855. In 1950 the production of bituminous coal in 1,000 tons was 512,000. How much more coal in 1,000 tons was produced in 1950? How many tons is this?
4. In a recent year 537,566 tons of zinc ore were produced in the United States. The following year 591,454 tons were produced. How many more tons were produced in the second year?
5. During one year the following numbers of 1,000 bushels of potatoes were marketed by the New England states: Maine, 61,705; New Hampshire, 980; Vermont, 1,092; Massachusetts, 2,816; Connecticut, 3,841; Rhode Island, 1,275. How many 1,000 bushels of potatoes were marketed in New England during the year? How many bushels is this?
6. In one year 917,778 tons of newsprint were produced. The following year 1,017,322 tons were produced. How many more tons were produced in the second year?
7. How many cars were manufactured in all during a 3-year period if the yearly productions were as follows: 3,909,270; 5,119,466; and 6,665,628?
8. How many farms were being cultivated in these three Middle Atlantic states in a certain year if there were 176,432 in New York, 28,873 in New Jersey, and 190,911 in Pennsylvania?
9. One year there were 70,009 miles of pipeline for petroleum in use in the United States. Twenty years later there were 111,615 miles of pipeline in use. How many more miles of pipeline were in use in the later year?



Adding and Subtracting Numbers of Dollars and Cents

Wheat is man's most important food crop. There are about thirty thousand varieties grown in the world, and the United States raises more than two hundred varieties.

1. A farmer sold his year's crop at the following prices: \$454.30, \$507.80, \$678.50, and \$563.20. How much did he receive in all?
   What do you do to find how much in all?

\$ 454.30
507.80
678.50
563.20
\$2203.80

Add the dollars and the cents just as you add other numbers. Keep the decimal points in line. Remember to write a dollar sign and a decimal point in the sum.

The farmer received \$2203.80 that year for his wheat crop.

2. The year before, the farmer had received \$1674.90 for his wheat. This was how much less than the amount he received above?
   What do you do to find how much less?

\$2203.80
1674.90
\$ 528.90

Subtract dollars and cents just as you subtract other numbers. Remember to write the dollar sign and the decimal point correctly in the difference.

Check your work by adding the difference to the subtrahend. The farmer had received \$528.90 less the year before.

Copy the following examples and add. Check your sums.

3.

\$.03	\$.08	\$.53	\$.87	\$3.74
.09	.07	.38	.43	7.05
.07	.09	.47	.65	2.81
.01	.06	.19	.98	.60
.04	.04	.34	.54	.39
.02	.05	.26	.76	.47

4.

\$5.27	\$4.13	\$7.87	\$25.83	\$53.46
.96	1.97	2.53	46.07	8.07
1.41	6.50	4.06	11.60	49.80
.73	3.31	8.42	30.59	2.75
8.65	9.08	6.25	24.72	4.61
.84	5.76	3.57		

5.

$75.08	$134.72	$631.80	$749.51	$2340.75
6.46	480.56	807.45	86.03	6802.19
.93	297.03	390.06	2.59	1126.48
4.72	561.28	266.63	.78	

Copy and subtract. Check your answers.

6.

$657.49	$420.06	$307.50	$700.00
368.69	273.58	268.75	486.35

7.

$176.51	$3876.57	$7345.60	$8046.70
87.83	1398.63	4068.75	5367.85

8.

$4300.00	$1642.70	$2804.52	$49531.55
3645.61	753.88	956.63	27846.75

9.

$65746.48	$75000.00	$13406.20	$37905.41
46809.69	68432.75	8685.73	8916.43

10. The Webers plan to picnic in their yard. Find the total cost of rolls at $.39, hamburger meat at $1.56, milk at $.42, paper plates at $.15, and watermelon at $.68.

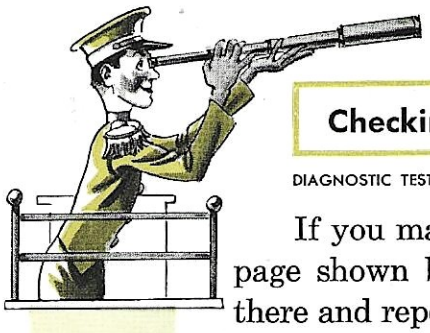
11. Mr. Weber has bought an outdoor grill for $29.95, a picnic table and benches for $34.88, and outdoor chairs for $9.95. What is the total cost?

12. An outdoor gym with swings and trapezes, usually priced at $44.50, is on special sale for $29.95. How much is saved by buying it at the sale?

13. Mrs. Weber wants a sandbox for Teddy, which costs $13.95, and a canvas pool costing $19.89. What would be the total cost?

14. Arthur looked at bicycles. He saw one priced at $37.95, and another at $29.99. How much more was one than the other?

15. Mr. Weber's new car will cost $2258.69. The dealer is allowing him $975 on his old car for this sale. How much money will Mr. Weber pay the dealer?



Checking Up on Addition and Subtraction

DIAGNOSTIC TEST ON PAGES 189–196

If you make mistakes in a row of examples, turn to the page shown beside the row. Study the explanation found there and repeat the practice. Then try the row on this page again.

Copy. Add down. Check by adding up.

189 1.

91	17	125	450	569
86	24	672	321	10
15	63	596	804	873
47	50	200	67	695
63	98	784	748	24
22	32	861	986	31

190 2.

5691	3284	6900	7865	8125
2840	615	1875	400	769
3786	473	5761	1974	52
4954	5796	7058	793	84
		4597	4659	1873

191 3.

32,561	45,775	63,195	57,643	76,845
65,786	9,809	986	18,209	5,937
10,800	78,624	2,000	24,874	198
			76,586	61,750

192 4.

531,896	125,000	115,956	486,275	786,349
120,500	673,750	72,000	19,863	13,675
698,425	239,898	98,724	54,758	108,960

Copy. Subtract. Check by adding.

193 5.

37,629	54,650	786,500	475,086
10,500	9,135	234,475	53,128

194 6.

6,586,375	5,425,000	2,986,480	5,875,649
1,402,150	2,150,000	179,625	98,375

Copy and add. Keep the decimal points in line.

7.

\$.03	\$3.69	\$10.50	\$356.25
.17	2.15	29.75	791.50
.25	.78	76.38	684.37
.09	6.90	5.09	573.82

196

Copy and subtract. Check your answers.

8.

\$7.98	\$29.95	\$356.75	\$28,654.32
5.75	18.50	189.98	7,328.65

196

Cattle Feed on the Grasslands

MULTIPLYING 5–PLACE BY 1–PLACE NUMBERS

Bob knows that grasslands are one of our most important natural resources. Millions of acres of pasture are needed each year for the cattle who produce our meat and dairy products.

Bob read that a Guernsey cow had been known to produce 26,672 pounds of milk in one year. He wondered how many pounds of milk this cow could produce in 3 years.

Bob thought, "To find the answer, I multiply 26,672 by 3."

First Bob multiplied the ones. $3 \times 2 = 6$. He wrote 6 in the ones place.

Next Bob multiplied the tens. $3 \times 7 = 21$. 21 tens = 2 hundreds 1 ten. He wrote 1 in the tens place and kept the 2 hundreds to add with the other hundreds.

Then he multiplied the hundreds. $3 \times 6 = 18$. He added the 2 hundreds he was keeping with the hundreds. $18 + 2 = 20$. 20 hundreds = 2 thousands 0 hundreds. He wrote 0 in the hundreds place and kept the 2 thousands to add with the other thousands.

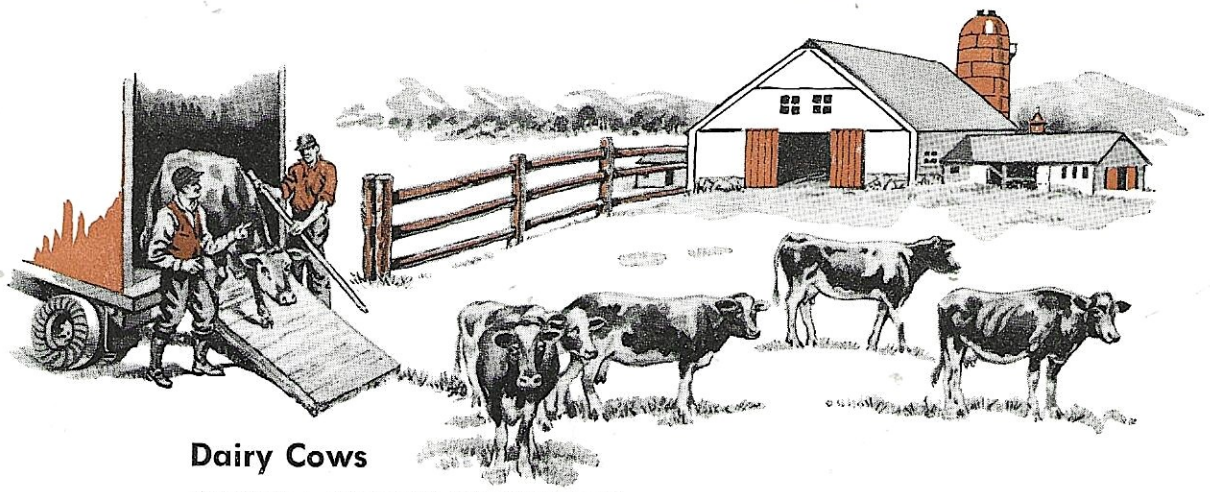
In the same way Bob multiplied the thousands and ten thousands. Bob found that the answer was 80,016. This cow could produce 80,016 pounds of milk in 3 years.

$$\begin{array}{r} 26{,}672 \\ \times\ \ 3 \\ \hline 80{,}016 \end{array}$$

Now turn the page.

Copy the examples below and multiply. Check by going over your work.

1.	28,560 × 7	25,490 × 6	54,923 × 4	61,570 × 5	73,651 × 8
2.	87,940 × 3	66,392 × 9	76,809 × 4	68,700 × 8	54,327 × 5
3.	98,164 × 2	24,589 × 8	10,487 × 9	95,437 × 7	90,864 × 5
4.	18,765 × 4	65,312 × 3	70,532 × 2	74,793 × 6	75,831 × 9
5.	13,698 × 6	19,743 × 7	54,897 × 6	94,382 × 8	20,906 × 7



Dairy Cows

MULTIPLYING 4– AND 5–PLACE BY 2–PLACE NUMBERS

Mr. Jones told Bob that he had just bought 15 Holstein cows. The cows had been brought to the farm in a large truck. Bob was thinking about the load on the truck. He wondered if the 15 cows would weigh 15 tons.

Mr. Jones said, "No, each cow weighs about 1500 pounds."

"Then the cows weigh less than a ton apiece," said Bob.

"A ton is 2000 pounds. Half a ton would be 1000 pounds, a quarter of a ton would be 500 pounds, so 1500 pounds would be three quarters of a ton. They each weigh about $\frac{3}{4}$ ton."

Bob decided to find out how much the 15 cows weighed. He multiplied 1500 by 15. His example is shown at the right. Study his example. Be sure you can do one like it.

```
 1500
   15
 7500
1500
22500
```

Remember that when you multiply by a two-place number you multiply by one number at a time, being careful to write the figures neatly and in the proper place. First multiply by 5 ones. 5 ones × 1500 = 7500 ones. Write this number with the last figure 0 directly below the 5 ones in the multiplier. Next multiply by 1 ten. 1 ten × 1500 = 1500 tens. Write this number with the last figure 0 directly below the 1 ten in the multiplier. Then add the partial products.

The answer is 22,500. The total weight of the 15 cows would be approximately 22,500 pounds.

Study the multiplication example at the right. Then copy it without the work. Multiply. Compare your results with the work in the book.

```
 7063
   45
35315
28252
317835
```

Copy and multiply. Check by going over your work.

1.	2453	5874	6923	8060	4509
	56	27	47	75	89
2.	8090	8517	9036	8714	6085
	64	78	92	34	83
3.	1200	1238	8569	3677	6672
	21	98	25	49	38
4.	84617	52938	93421	24739	65790
	65	46	37	85	67

5. Multiply 15 × 2000 to find how many pounds 15 tons would be. How much less than 15 tons did Mr. Jones's cows weigh?

Mining Zinc Ore

MULTIPLYING 4–PLACE BY 3–PLACE NUMBERS

In a recent year the average daily production of zinc ore in the United States was 2,276 tons. How many tons of zinc ore will be mined at this rate during a year of 312 working days?

To find the answer, multiply 2276 by 312.

```
 2276
  312
 4552
2276
6828
710112
```

First multiply by ones. 2 ones × 2276 = 4552 ones. Write 4552, the first partial product, with 2 in ones place.

Next multiply by tens. 1 ten × 2276 = 2276 tens. Write 2276, the second partial product, with 6 in tens place.

Then multiply by hundreds. 3 hundreds × 2276 = 6828 hundreds. Write 6828, the third partial product, with 8 in hundreds place.

Add the partial products. Their sum is 710,112.

710,112 tons of zinc ore will be mined in 312 working days.

Copy and multiply. Be sure to write each figure neatly and in its proper place. Check by going over your work.

1.	3424	5132	6342	8311	7403
	212	323	121	233	122
2.	3246	6375	5407	9652	1345
	234	326	465	347	289
3.	3670	8057	3408	7060	8004
	543	628	287	465	455

If you can do the examples above correctly, you can do almost any multiplication example with whole numbers. Use what you know about place value and about how to multiply by ones, tens, and hundreds to help you figure out how to multiply by even larger numbers. These examples take a lot of room. Copy only one at a time, and leave space to the left of it.

4.	2012	3265	31486	62801
	1234	2113	8975	71623

Zeros in Multiplication

1. Suppose you want to multiply 3600 by 30. Here are some ways in which you may do this example.

3600	3600	3600	3600
3 tens	30	30	30
10800 tens	108000	0000	108000
= 108000		10800	
		108000	

How are these ways alike? How are they different?

2. Multiply 1400 by 600. Here are some ways in which you may do this example.

1400	1400	1400	1400
6 hundreds	600	600	600
8400 hundreds	840000	0000	840000
= 840000		0000	
		8400	
		840000	

In the third way, which of the zeros could have been left out? How can you tell which zeros must be written?

Do the following examples in the way that makes the most sense to you. Copy and multiply. Check your work.

3. 1300	2400	3700	4600	6700
40	30	70	80	50
4. 3000	5000	8000	6000	7000
70	90	40	80	60
5. 1400	2300	4200	3600	5800
300	200	700	400	500
6. 3400	4700	2600	4500	5500
400	300	800	900	600

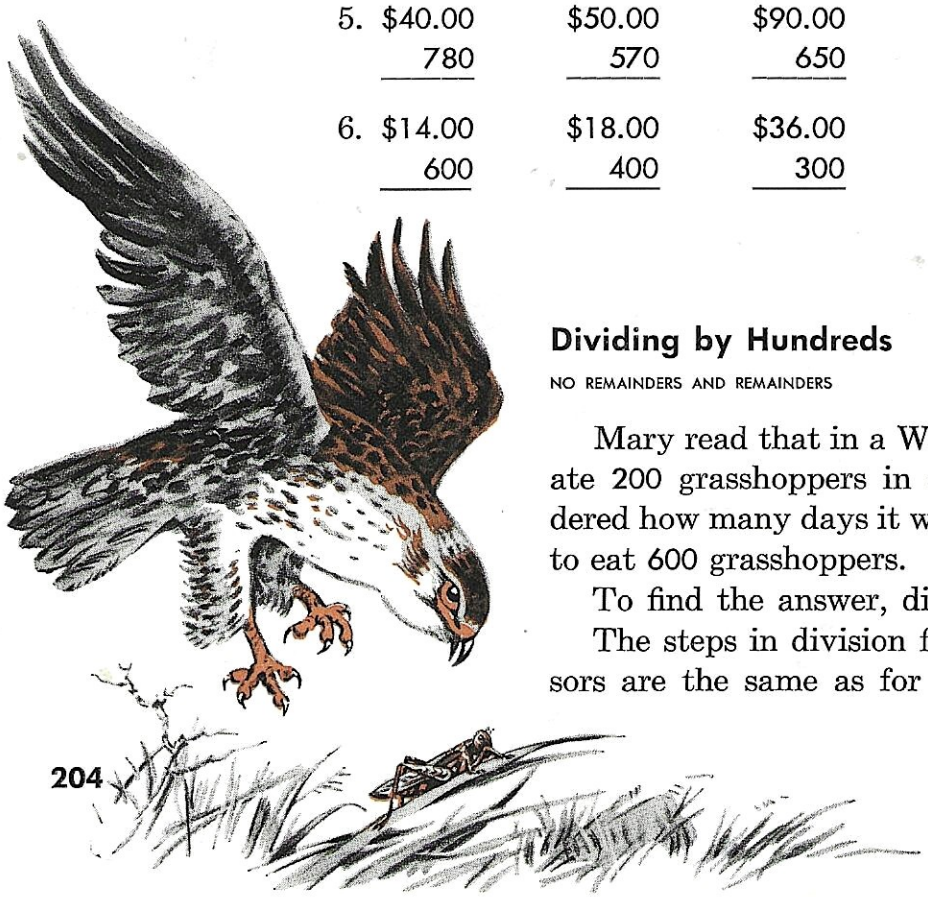
Multiplying Numbers of Dollars and Cents

$$\begin{array}{r} \$6\ 3.2\ 5 \\ \times\quad 1\ 2\ 5 \\ \hline 3\ 1\ 6\ 2\ 5 \\ 1\ 2\ 6\ 5\ 0 \\ 6\ 3\ 2\ 5 \\ \hline \$7\ 9\ 0\ 6.2\ 5 \end{array}$$

Study the example at the left. You multiply dollars and cents just as you multiply other numbers. The answer must have a dollar sign and a decimal point to show that it means dollars and cents.

Copy and multiply. Check by going over your work.

1.	\$12.45	\$23.81	\$46.37	\$53.25	\$82.48
	234	145	326	257	265
2.	\$30.20	\$40.60	\$50.70	\$20.50	\$80.70
	216	138	357	645	468
3.	\$20.03	\$60.04	\$90.06	\$40.05	\$80.05
	123	346	236	467	354
4.	\$24.00	\$36.00	\$43.00	\$52.00	\$68.00
	260	370	450	380	650
5.	\$40.00	\$50.00	\$90.00	\$60.00	\$70.00
	780	570	650	450	860
6.	\$14.00	\$18.00	\$36.00	\$48.00	\$40.00
	600	400	300	500	800



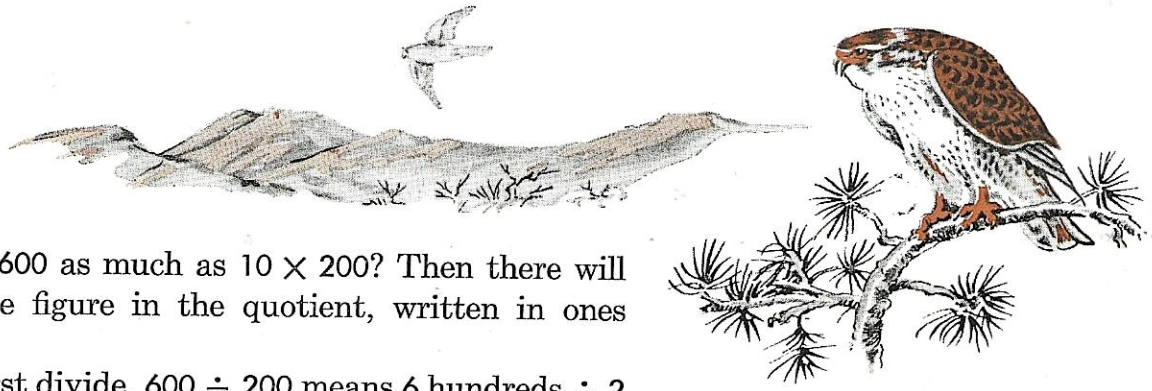
Dividing by Hundreds

NO REMAINDERS AND REMAINDERS

Mary read that in a Western state a hawk ate 200 grasshoppers in one day. She wondered how many days it would take the hawk to eat 600 grasshoppers.

To find the answer, divide 600 by 200.

The steps in division for three-place divisors are the same as for two-place divisors.



Is 600 as much as 10×200? Then there will be one figure in the quotient, written in ones place.

First divide. $600 \div 200$ means 6 hundreds $\div$ 2 hundreds. There are as many 2 hundreds in 6 hundreds as there are 2's in 6. $6 \div 2 = 3$. Where do you write the 3 in the quotient? Next multiply. $3 \times 200 = 600$. Compare. 600 is not larger than 600, so subtract. There is no remainder.

The hawk will eat 600 grasshoppers in 3 days.

$$\begin{array}{r} 3 \\ 200\overline{)600} \\ \underline{600} \end{array}$$

Check

$$\begin{array}{r} 200 \\ \times 3 \\ \hline 600 \end{array}$$

Study the example at the right. Is 1315 as much as 10×300? Then how many figures will be in the quotient?

To divide 1315 by 300, think of 1315 as 13 hundreds 1 ten 5 ones. There are about as many 3 hundreds in 13 hundreds 1 ten 5 ones as there are 3's in 13. $13 \div 3 = 4$ with a remainder. Where do you write the 4 in the quotient? Now multiply and compare. $4 \times 300 =$ ___?___. 1200 is not larger than 1315. Subtract and compare. 115 is smaller than 300. There is no number to bring down. 115 is the remainder.

The answer is 4 r115. Does it check?

$$\begin{array}{r} 4 \text{ r}115 \\ 300\overline{)1315} \\ \underline{1200} \\ 115 \end{array}$$

Check

$$\begin{array}{r} 300 \\ \times 4 \\ \hline 1200 \\ +115 \\ \hline 1315 \end{array}$$

Copy and divide. Express any remainders as shown in the example above. Check each answer.

1. $200\overline{)400}$ $400\overline{)800}$ $500\overline{)1500}$ $300\overline{)1200}$ $600\overline{)1800}$
2. $700\overline{)2100}$ $900\overline{)3600}$ $800\overline{)5600}$ $400\overline{)2800}$ $500\overline{)4000}$
3. $300\overline{)925}$ $200\overline{)710}$ $400\overline{)1830}$ $500\overline{)3600}$ $700\overline{)4327}$
4. $600\overline{)2636}$ $800\overline{)3300}$ $900\overline{)4705}$ $200\overline{)1910}$ $400\overline{)3000}$
5. $500\overline{)4630}$ $700\overline{)5032}$ $600\overline{)2075}$ $900\overline{)5475}$ $800\overline{)4264}$

Dividing by Three-Place Numbers

ONE-PLACE QUOTIENTS

Do you know how to find the number of square feet in a larger number of square inches? What process will you use? What number will you divide by?

```
      2
144)288
    288

Check
  144
 ×2
  288
```

1. To find how many square feet there are in 288 square inches, Jack divided as shown at the left. How many figures will there be in the quotient? How can you tell? Does his answer check? Jack found that there are 2 square feet in 288 square inches.

```
      5
144)576
    720

      4
144)576
    576
```

2. To find the number of square feet in 576 square inches, Jack thought, "There are about as many 144's in 576 as there are 1's in 5." He tried 5. Is 5 the correct quotient figure? Jack saw that 5 is too large, so he tried the next smaller number, 4. What is the correct quotient? Check Jack's work.

```
      5 r7
342)1717
   1710
      7
```

3. Study the example at the left. How can you tell that the quotient will have one figure? Why do you have to use the first *two* figures in the dividend (17) to divide by the first figure in the divisor (3) in estimating the quotient in this example? Check the example.

Copy, divide, and check. Use the first figure in the divisor and the first one or two figures in the dividend in estimating the quotient, just as you did in dividing by two-place numbers.

4.	121)484	234)702	321)963	412)824
5.	144)432	426)852	347)694	258)774
6.	212)944	335)1061	246)984	219)897
7.	739)2835	127)898	256)819	834)2604
8.	425)1467	329)1560	456)495	709)1556

Dividing by Three-Place Numbers

TWO-PLACE QUOTIENTS

Tim learned that Mr. Thomas harvested 27,648 pounds of cotton in one of his fields. With an average weight of 512 pounds per bale, how many bales of cotton did he harvest?

To find the answer, divide 27648 by 512.

How many figures will there be in the quotient? How can you tell? Where will you write the first quotient figure?

To find the first quotient figure, divide 2764 tens by 512.

Step 1. *Divide.* There are about as many 512's in 2764 as there are 5's in 27. Write 5 in tens place in the quotient.

Step 2. *Multiply and compare.* 5×512 is 2560. 2560 is not larger than 2764.

Step 3. *Subtract and compare.* 2764 — 2560 is 204. 204 is less than 512.

Step 4. *Bring down.* Bring down 8 ones from the dividend.

Now follow the same steps for dividing the ones. You have 2048 ones to divide by 512.

The quotient is 54. Mr. Thomas harvested 54 bales of cotton.

```
      54
512)27648
    2560
     2048
     2048

Check
     512
    ×54
    2048
   2560
  27648
```

1. Study the example at the right. How many figures will be in the quotient? Why? Where will you write the first quotient figure? To find the first quotient figure, divide 1146 tens by 224. Explain how you finish the example. The answer is 51 r44. Notice how the check is done. In checking division examples, multiply the easier way.

```
      51 r44
224)11468
    1120
     268
     224
      44

Check
     224
    ×51
     224
   1120
  11424
    +44
  11468
```

Copy, divide, and check.

2. 123)3864 105)4395 315)7425 856)35952

3. 436)9326 248)7688 234)2687 529)16407

4. 634)32345 621)16149 315)6696 534)6408

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–5)

1. Tell which of the following is a proper fraction, an improper fraction, a decimal, a mixed number: $2\frac{1}{2}$; .5; $\frac{3}{7}$; $\frac{6}{5}$.

2. Round the following amounts to the nearest ten cents:

   \$.47 \$.52 \$.85 \$.68 \$.14

3. Give the results of the following: $\frac{3}{4}$ of 36; $\frac{4}{5}$ of 75; $\frac{7}{8}$ of 56.

4. Is the product of 234×52 the same as the product of 52×234? Which order of multiplier and multiplicand is easier to use?

5. Find the sums. Be sure your answers are in lowest terms.

$\frac{1}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{2}{3}$	$1\frac{1}{4}$	$3\frac{4}{5}$
$\frac{2}{7}$	$\frac{2}{8}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{4}{9}$	$2\frac{1}{3}$	$4\frac{1}{2}$

6. Subtract. Be sure your answers are in lowest terms.

$\frac{2}{3}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{7}{9}$	$1\frac{1}{2}$	1	$7\frac{1}{4}$
$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{5}$	$2\frac{1}{3}$

Copy and fill in the missing numbers.

7. 36 in. = _?_ ft. 9 yd. = _?_ ft. 2 mi. = _?_ ft.

8. 2 c. = _?_ tbsp. 4 pk. = _?_ bu. 32 oz. = _?_ lb.

9. $\frac{1}{4}$ T. = _?_ lb. 48 hr. = _?_ da. 30 min. = _?_ hr.

10. 9 sq. ft. = _?_ sq. yd. $\frac{1}{2}$ sq. ft. = _?_ sq. in. $\frac{1}{2}$ mi. = _?_ ft.

Add. Keep the decimal points in line.

11.

.5	.7	3.1	4.0	6.4	\$3.50	\$5.04
.3	.6	2.7	1.9	.8	1.79	2.10

Subtract and check.

12.

.7	.4	8.6	5.4	6.0	\$4.79	\$5.00
.1	.2	2.6	3.9	.3	2.25	1.37

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–5)

Read each problem carefully. Decide whether to add, subtract, multiply, or divide. Then find the answer.

1. An airline company advertises a week's trip from New York to California for $252.35, with a down payment of $25.35. How much would be left to pay?
2. In 1492 it took Columbus 70 days to sail from Spain to North America. He averaged 67 miles a day. How many miles did he travel?
3. The Miller family drove 172.4 miles on Monday, 228.5 on Tuesday, and 97.8 on Wednesday. How many miles did they drive during these three days?
4. The distance between two Eastern cities is 840 miles. A plane takes from 7:00 A.M. to 11:00 A.M. to make the trip. What is its average speed per hour?
5. In the past three weeks, Mrs. Smith bought beef kidneys for the family's pet cat. The kidneys weighed $14\frac{1}{2}$ ounces, $13\frac{3}{4}$ ounces, and $13\frac{1}{4}$ ounces. How much did the beef kidneys Mrs. Smith bought weigh all together?
6. A truck is carrying 3,848 pounds of peaches. At 74 pounds to a bushel, how many bushels of peaches is the truck carrying?
7. In 1956 an average of 7 babies were born each minute in the United States. How many was that in an hour? In a day?
8. Typewriter paper is 11 inches long and $8\frac{1}{2}$ inches wide. How much longer is the paper than its width?
9. On his way to the park, Jack rode 1.8 miles on his bicycle. On his return he rode 2.3 miles. How much farther was his return trip?
10. The first game of the season was attended by 368 persons; the second by 422. Find the average attendance for the two games.
11. A football field is shaped like a rectangle. It is 360 feet long and 100 feet wide. How many square feet are there in its area? How many square yards?
12. A softball diamond is shaped like a square. It is 60 feet on a side. Find its area in square feet. In square yards.

Estimating Quotient Figures

SOME APPARENT QUOTIENTS INCORRECT

Dividing by three-place numbers is like dividing by two-place numbers except that you have to think of rounding the divisor to the nearest hundred instead of to the nearest ten.

```
      75 r121
395)29746
   2765
   2096
   1975
    121

Check
    395
   ×75
   1975
  2765
  29625
  +121
  29746
```

1. Divide 29746 by 395. Study the example at the left. How can you tell that the quotient will have two figures?

Look at the divisor. Is 395 closer to 300 or to 400? Then, to estimate the first quotient figure, think 4's in 29, _?_. Multiply and compare. $7 \times 395 =$ _?_. Since 2765 is not more than 2974, subtract and compare. 209 is less than 395, so bring down 6 ones from the dividend. Now divide again. 4's in 20, _?_. Multiply and compare. $5 \times 395 =$ _?_. 1975 is less than 2096, so subtract and compare. 121 is less than 395. 121 is the remainder.

2. Study these three examples. Is 462 closer to 400 or 500? Is 872 closer to 800 or 900? Is 783 closer to 700 or 800? When the second figure of the divisor is 6, 7, 8, or 9, it usually saves time to raise the first figure by 1 to use as the number you think with when you divide. Tell how you would think in dividing each of these examples.

```
      38 r313          55 r338          44 r766
462)17869        872)48298        783)35218
   1386             4360             3132
   4009             4698             3898
   3696             4360             3132
    313              338              766
```

Copy, divide, and check.

3. $390\overline{)22200}$ $864\overline{)84245}$ $295\overline{)20184}$ $898\overline{)24344}$

You know that sometimes more than one trial is needed to find the correct quotient figure. This is review. Study the following examples.

4. The second figure of the divisor is 9, so raise the first figure by 1 and use 3 (hundreds) to divide by. To find the first figure in the quotient, think: 3's in 14, 4. Is 4 the correct figure for tens place in the quotient? What numbers do you compare to find out? To find the second figure in the quotient, think: 3's in 26, 8. Is 8 the correct quotient figure? How can you tell? Is 8 too large or too small? What do you do now? Finish the example.

```
   48
295)14463
  1180
  2663
  2360
   303
```

5. The second figure of the divisor is 3, so use the first figure as it is, 7 (hundreds), to divide by. To find the first figure in the quotient, think: 7's in 57, 8. Is 8 the correct figure for tens place in the quotient? How can you tell? Is 8 too large or too small? What do you do now? Finish the example.

```
    8
732)57618
  5856
```

6. Sometimes multiplying mentally the first two figures of the divisor by the trial quotient will tell you whether the trial quotient is too large. Would this have helped with example 5?

7. What is the largest figure you can have in any place in the quotient?

Copy, divide, and check.

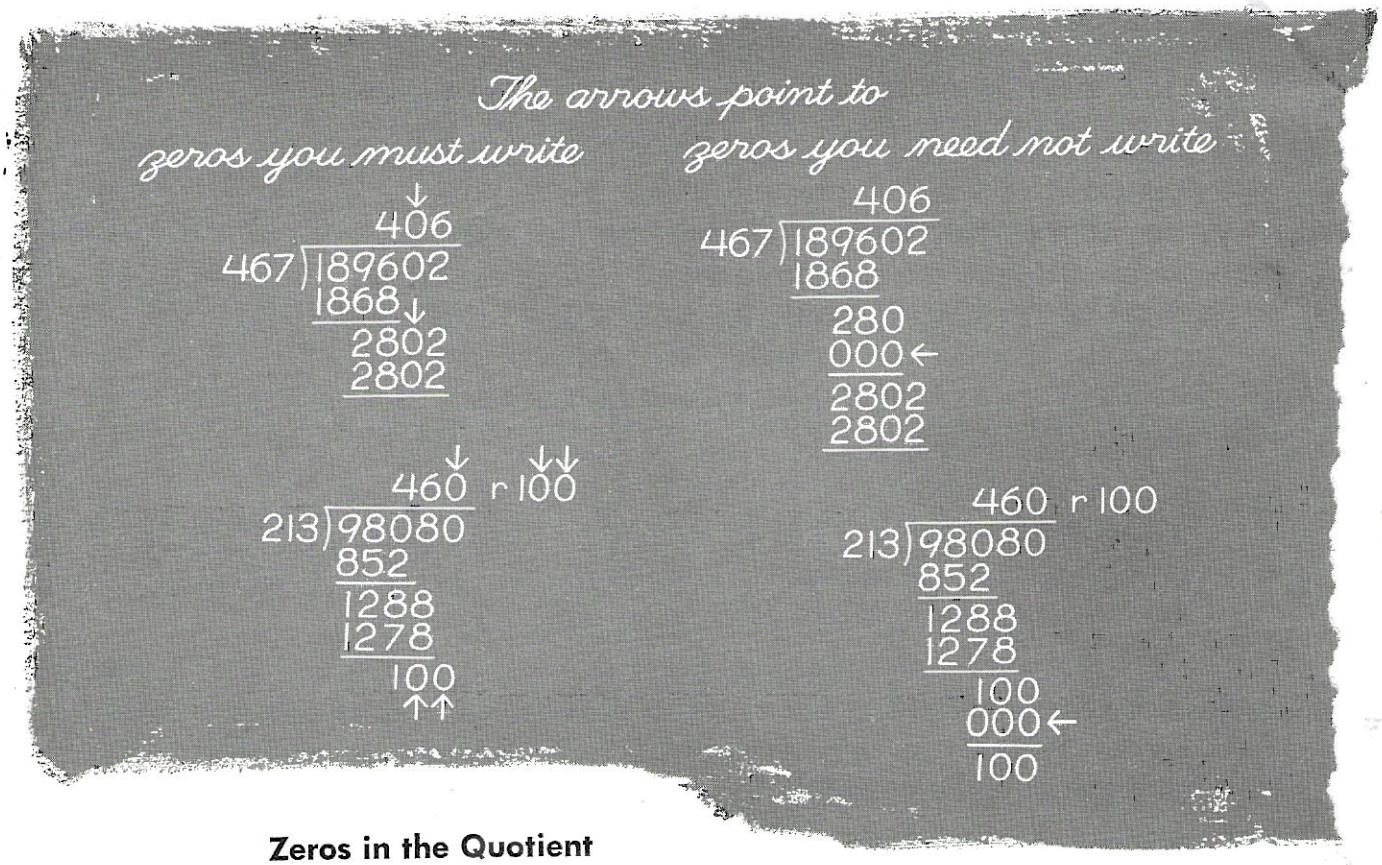
8. $126\overline{)4158}$ $273\overline{)24305}$ $349\overline{)21289}$ $813\overline{)18654}$

9. $254\overline{)4064}$ $284\overline{)18744}$ $768\overline{)30000}$ $142\overline{)13925}$

The examples which follow have more than two figures in the quotient. Copy them carefully one at a time so that you will surely have enough room for your work. Decide where to write the first figure in the quotient. Write all your figures neatly and in their proper places so that you do not get mixed up. If you can do these examples correctly without help, you can do almost any long division. Divide and check.

10. $856\overline{)360376}$ $321\overline{)92127}$ $592\overline{)204240}$

11. $248\overline{)768312}$ $436\overline{)932604}$ $821\overline{)999764}$



Zeros in the Quotient

MIDDLE AND TERMINAL ZEROS

The pupils in Miss White's room were working with division examples that have zeros in the quotient. They were talking about how to decide when you must write a zero and when you may leave it out.

Tom said, "You must write a zero when it is needed to make a number mean what it is supposed to mean. For instance, if the number is either 406 or 460 and you leave out the zero, making 46, you would have the same result for two very different numbers, and furthermore, your number would mean less than one hundred when it is supposed to mean more than four hundred."

"That's right," said Judy. "The times you don't have to write zeros are when they don't make any difference in the answer. When you have a whole row of zeros to be added or subtracted is one time you don't have to write them, because you come out with the number you had at first anyway."

Study the following examples. Notice how carefully the figures are kept in line. Notice that after the first figure in the quotient is written, there is a quotient figure above each figure in the dividend; the last quotient figure is written directly above the last figure in the dividend. Copy the examples without the work. Do the examples and compare your work with the work in the book.

1.
$$\begin{array}{r} 307 \\ 825\overline{)253275} \\ 2475 \\ \hline 5775 \\ 5775 \\ \hline \end{array}$$

$$\begin{array}{r} 406\ r17 \\ 716\overline{)290713} \\ 2864 \\ \hline 4313 \\ 4296 \\ \hline 17 \end{array}$$

$$\begin{array}{r} 630\ r167 \\ 219\overline{)138137} \\ 1314 \\ \hline 673 \\ 657 \\ \hline 167 \end{array}$$

2.
$$\begin{array}{r} 250 \\ 913\overline{)228250} \\ 1826 \\ \hline 4565 \\ 4565 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 400\ r46 \\ 164\overline{)65646} \\ 656 \\ \hline 46 \end{array}$$

$$\begin{array}{r} 700 \\ 345\overline{)241500} \\ 2415 \\ \hline 00 \end{array}$$

Copy, divide, and check.

3.	$254\overline{)103236}$	$384\overline{)78720}$	$481\overline{)291069}$	$248\overline{)174840}$
4.	$347\overline{)105256}$	$536\overline{)270194}$	$327\overline{)131267}$	$432\overline{)8903}$
5.	$461\overline{)49788}$	$625\overline{)65659}$	$525\overline{)315921}$	$253\overline{)177859}$
6.	$626\overline{)65730}$	$476\overline{)335580}$	$245\overline{)220995}$	$342\overline{)206584}$
7.	$364\overline{)101920}$	$294\overline{)135378}$	$198\overline{)63360}$	$523\overline{)349985}$
8.	$419\overline{)217880}$	$634\overline{)463145}$	$256\overline{)207568}$	$715\overline{)164450}$
9.	$365\overline{)332150}$	$924\overline{)138742}$	$373\overline{)164375}$	$581\overline{)400890}$
10.	$145\overline{)84215}$	$615\overline{)264450}$	$432\overline{)328578}$	$951\overline{)475595}$
11.	$906\overline{)724985}$	$468\overline{)280927}$	$176\overline{)123315}$	$452\overline{)406800}$

Dividing Numbers of Dollars and Cents

THREE-PLACE DIVISORS

Dividing amounts of money is the same as other division except that you have to think carefully how to write the answer.

125)$57.75

1. Think about the example at the left. Will the quotient mean money? If there is a remainder, will that be money? If the remainder is less than 100, will it stand for a certain number of cents? In what two ways can you write a number of cents? If the remainder is more than 100, will it stand for an amount of money more than a dollar? Now do the example. Your answer should be $.46 r$.25, or the same numbers written with cents signs.

Study the following examples. Then copy them without the work and do them. Compare your work with the work in the book.

2.

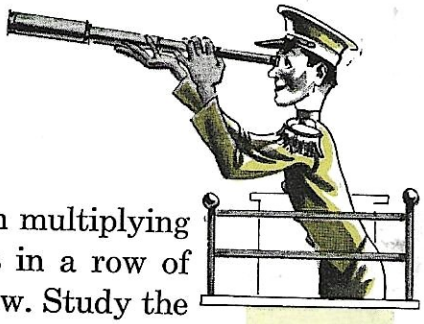
$.15	$1.06 r45¢	$7.04 r$1.35
325)$48.75	275)$291.95	635)$4471.75
32 5	275	4445
16 25	16 95	26 75
16 25	16 50	25 40
	45	1 35

Divide and check.

3.	235)$32.90	450)$103.50	525)$236.25	655)$196.50
4.	175)$45.75	542)$287.88	732)$468.98	850)$637.62
5.	345)$1035.00	264)$1848.50	164)$656.75	531)$4249.57
6.	132)$271.60	254)$777.24	467)$485.68	825)$3316.50
7.	248)$1004.90	480)$2913.85	905)$4607.55	635)$4470.75
8.	219)$1752.73	687)$4814.85	395)$1582.56	931)$2796.62
9.	175)$350.50	425)$1276.78	682)$3413.45	452)$4069.50

Checking Up on Multiplication and Division

DIAGNOSTIC TEST ON PAGES 199–214



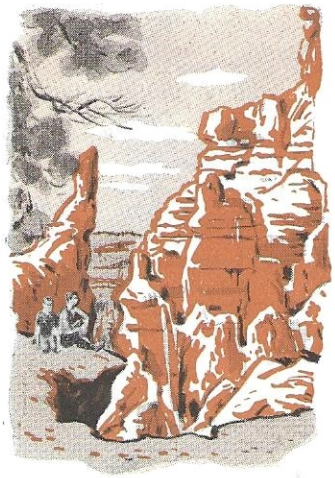
This is a test to find out if you need more help in multiplying and dividing larger numbers. If you make mistakes in a row of examples, turn to the page shown at the end of the row. Study the explanation and repeat the practice. Then try the row on this page again.

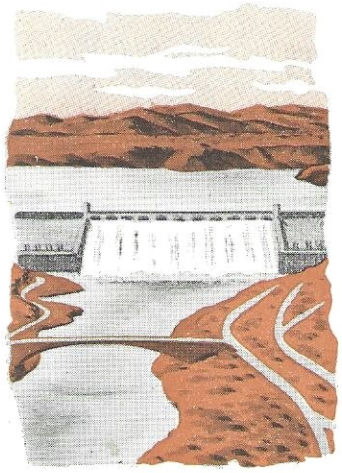
Copy. Multiply. Check by going over your work.

1.	43,561 × 2	37,892 × 6	56,400 × 5	50,376 × 8	60,509 × 7	199
2.	2634 × 13	5789 × 64	8075 × 97	1798 × 58	7020 × 49	201
3.	2145 × 312	6987 × 543	7025 × 678	8060 × 954	9007 × 678	202
4.	2100 × 30	5700 × 40	8000 × 70	6400 × 500	9500 × 800	203
5.	\$12.25 × 342	\$23.81 × 165	\$25.20 × 976	\$60.05 × 860	\$30.00 × 700	204

Copy. Divide. Check your work.

6.	$300\overline{)900}$	$400\overline{)1600}$	$500\overline{)2100}$	$700\overline{)3785}$	$600\overline{)4415}$	205
7.	$212\overline{)848}$	$432\overline{)1296}$	$213\overline{)868}$	$418\overline{)1792}$	$144\overline{)596}$	206
8.	$214\overline{)7490}$	$338\overline{)16192}$	$825\overline{)59475}$	$719\overline{)11840}$	$443\overline{)32650}$	207
9.	$132\overline{)5696}$	$138\overline{)7344}$	$290\overline{)5245}$	$264\overline{)21975}$	$312\overline{)7822}$	210
10.	$534\overline{)107868}$	$987\overline{)494500}$	$642\overline{)224775}$	$798\overline{)478800}$		213
11.	$215\overline{)\$53.75}$	$426\overline{)\$1282.26}$	$352\overline{)\$2126.28}$	$405\overline{)\$797.85}$		214





Problems on Many Topics

In order to solve problems, you have to know not only *how* to add, subtract, multiply, and divide, but *when* to do them. *Add, subtract, multiply,* and *divide* are action words. Review for yourself the action that each of them describes. Read each problem carefully. Then use the action involved in the story of the problem to help you decide which process to use.

1. In Bryce Canyon there are 36,010 acres; in Zion Canyon there are 94,271 acres; and in Cedar Breaks National Monument there are 6,172 acres. What is the combined area of these three places?
2. What is the total area of the National Parks and Monuments in South Dakota? There are 123,492 acres in the Badlands; 320 acres in Fossil Cycad; 1,275 acres in Jewel Cave; and 27,886 acres in Wind Cave.
3. In early days industry shipped cement in barrels holding 376 pounds. Now industry ships cement in bags weighing 94 pounds. How many more pounds were shipped in a barrel than in a bag?
4. What is the total area of the following National Recreational Areas of the United States? Cape Hatteras has 30,000 acres; Coulee Dam has 98,500 acres; Lake Mead has 1,899,728 acres; Millerton Lake has 11,605 acres; and Shadow Mountain has 10,231 acres.
5. Nature takes at least 500 years to make one inch of soil. How many years does it take nature to make 7 inches of soil?
6. If 5 Holstein cows weighed approximately 7,500 pounds, about how much would one Holstein cow weigh?
7. One bag of cement weighs 94 pounds. How much would 12 bags of cement weigh?

8. A hen lays about 180 eggs a year. At this rate how many eggs would 25 hens lay in a year?

9. In 1870 there were 31,082,000 cattle in the United States. In 1950 there were 77,963,000 cattle. How many more cattle were there in 1950?

10. How many horses were being used in all by the following five states in 1950 if 352,000 were used in Texas; 347,000 in Missouri; 264,000 in Minnesota; 242,000 in Iowa; and 236,000 in Nebraska?

11. A railroad tank car holds 210 barrels of oil. If an oil train is hauling an amount equal to 12,180 barrels, how many cars are on the train?

12. In a 24-hour period a large blast furnace will smelt as much as 1,500 tons of iron. How many tons can it smelt in one hour?

13. A queen bee can lay 2,500 eggs a day. How many eggs can 14 queen bees lay in a day?

14. A certain Kansas farm has pasture land valued at $63.25 an acre. How much would 125 acres of this land cost?

15. On a farm of 145 acres, a soil conservation expert estimated that over a period of years rains had washed out about 43,500 tons of topsoil. In this period, how many tons of topsoil per acre had been lost?

16. In 1900 there were 3,265,000 mules on farms. In 1950 there were 2,149,000 mules on farms. How many fewer mules on farms were there in 1950 than in 1900?

17. In a recent year there were 276,000 mules on farms in Mississippi. There were 139,000 mules on farms in Texas. How many more mules were on farms in Mississippi than in Texas?

What Have You Learned about Large Numbers?

CHAPTER TEST ON MEANINGS AND SKILLS

Copy the following statements, filling in the blanks.

1. The number 1,265,000 is read one _?_, two hundred sixty-five _?_. It is a _?_-place number.
2. To add large whole numbers, start at the _?_. First add the _?_, then the _?_, then the _?_, and so on.

Copy the following examples. Beside each number, write the word from the following list which describes the number.

addend *sum* *product* *quotient* *difference* *minuend*
divisor *multiplicand* *dividend* *multiplier* *subtrahend*

3. $\begin{array}{r} 486 \\ +15 \\ \hline 501 \end{array}$

4. $\begin{array}{r} 564 \\ -239 \\ \hline 325 \end{array}$

5. $10\overline{)250}$

6. $\begin{array}{r} 32 \\ \times 4 \\ \hline 128 \end{array}$

Copy. Add down. Check by adding up.

7.

76	223	5800	47,563	857,439
81	572	2785	28,308	25,765
30	796	5769	34,079	408,970
47	400	7048	86,586	
53	487	4596		
62	901			

Copy and subtract. Check by going over your work.

8.

39,725	63,540	476,056	$354.65	$37,546.32
12,807	9,137	58,408	167.98	9,460.65

Copy and multiply. Check by going over your work.

9.

54,892	8065	3245	5700	$35.40
8	97	432	80	654

Copy and divide. Check by going over your work.

10. $500\overline{)2200}$ $432\overline{)1298}$ $368\overline{)16198}$ $251\overline{)76309}$ $405\overline{)\$801.90}$

What Have You Learned about Large Numbers?

CHAPTER TEST ON PROBLEM SOLVING

1. In one large city the daily sales of one newspaper were 230,801 and of another newspaper 195,936. What was the difference in the daily sales of these newspapers?

2. The area in square miles of Illinois is 55,935, Indiana 36,205, Iowa 56,045, and Kansas 82,108. What is the total area of these four states?

3. Nebraska has an area of 77,227 square miles. Pennsylvania has an area of 45,333 square miles. How much larger is the area of Nebraska than the area of Pennsylvania?

4. A railroad to a large city carries an average of 56,784 persons daily. At this rate, how many persons does it carry in 5 days?

5. There are 500 sheets of paper in one package. How many packages can be made from 3,500 sheets of paper?

6. Mr. Johnson bought a house for $24,765.00. One year later he sold it for $26,540.00. How much money did he gain in selling the house?

7. The Parent-Teacher Association received $339.50 from dues, $143.75 from a play, and $105.25 from a food sale. What was the total amount received?

8. Mr. Adams earns $24.70 a day. How much does he earn in 261 working days?

9. An airplane travels at an average rate of 253 miles an hour. How many hours will it take this airplane to travel 1,518 miles?

10. Sally earns 75 cents an hour for baby-sitting. On Saturday she baby-sat for a neighbor from 8:45 P.M. to 11:45 P.M. and on Sunday from 8:00 A.M. to 1:00 P.M. How much money did she earn in all?

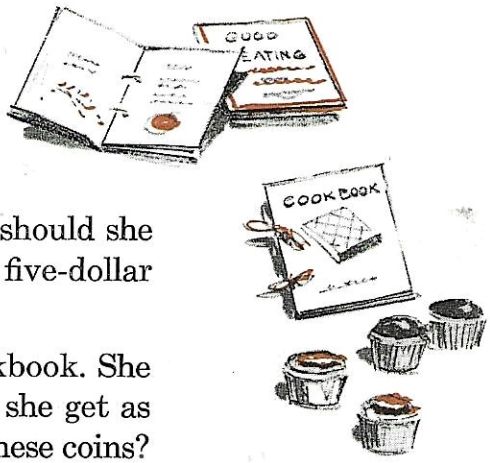
CHAPTER 7

The Cooking Project

The sixth grade wanted to make some money for a trip to the science museum. They decided to have a cooking project. Each pupil brought in a favorite recipe. The recipes were made into a cookbook. The class made 150 copies of the cookbook to sell. They had fun making the cookbooks, drawing pictures for them, and using the recipes themselves. Sometimes they ate the things they made. Sometimes they sold them and kept the money for their trip.

The picture shows one of their sales. Look at the picture when you want to find the prices you need to answer the questions below.

Multiplication of Fractions



1. Miss White bought 5 cookbooks. How much should she pay? How much change should she get from a five-dollar bill?
2. Tom is selling cookbooks. Ann bought 1 cookbook. She gave Tom a one-dollar bill. What coins might she get as change? What fraction of a dollar is each of these coins?
3. On Monday Tom collected $4.68. How many cookbooks had he sold?
4. It takes 20 whole sheets of paper for each cookbook. The paper comes 500 sheets to a package. How many packages of paper did the class use for 150 cookbooks?
5. Jane bought 4 pieces of fudge. How much did she spend?
6. How much would $\frac{1}{2}$ pound of fudge cost?

When a number is expressed so that any fraction in it is a proper fraction in lowest terms, the number is said to be in *simplest form*. In simplest form, $\frac{9}{12}$ would be $\frac{3}{4}$, $\frac{12}{6}$ would be 2, and $\frac{20}{16}$ would be $1\frac{1}{4}$.

For each question below, give your answer in simplest form.

7. Betty bought 8 cupcakes. What part of a dozen did she buy?
8. Jim bought $\frac{1}{2}$ pound of fudge. What part of a dollar did he spend?
9. Betty says that $\frac{36}{12}$ can tell how many dozen make 36. What is Betty's answer?
10. If you bought 18 cupcakes, how many dozen cupcakes would you have?

Are You Ready for Multiplication of Fractions?

READINESS TEST

If you can do the work on this page, you are ready for multiplication of fractions.

75 1. In the fraction $\frac{2}{5}$, the numerator is _?_ and the denominator is _?_.

83 2. Which of these fractions are expressed in lowest terms?

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{2}{6}$ $\frac{3}{8}$ $\frac{3}{9}$ $\frac{2}{5}$ $\frac{2}{10}$ $\frac{3}{12}$ $\frac{6}{9}$ $\frac{3}{10}$

49 3. Which of the following divide evenly by 2? By 3? By 5?

8 12 9 15 10 6 3 4 14 17

83 4. Change to lowest terms.

$\frac{4}{6}$ $\frac{3}{9}$ $\frac{2}{10}$ $\frac{3}{12}$ $\frac{7}{14}$ $\frac{4}{12}$ $\frac{5}{20}$ $\frac{14}{21}$ $\frac{6}{27}$ $\frac{10}{25}$

88 5. Which of the following are improper fractions? Which are mixed numbers?

$\frac{1}{3}$ $\frac{3}{3}$ $\frac{5}{3}$ $\frac{7}{8}$ $\frac{9}{8}$ $1\frac{7}{12}$ 14 $25\frac{1}{2}$ $\frac{8}{9}$ $3\frac{1}{4}$

221 6. Which of the following are in simplest form?

$2\frac{1}{2}$ $1\frac{3}{6}$ $\frac{7}{3}$ $4\frac{3}{9}$ $7\frac{5}{8}$ $\frac{9}{4}$ $\frac{5}{5}$ $3\frac{10}{12}$ $6\frac{5}{6}$ $2\frac{7}{15}$

221 7. Express in simplest form.

$\frac{3}{2}$ $\frac{8}{6}$ $\frac{8}{8}$ $\frac{5}{4}$ $\frac{18}{4}$ $\frac{12}{12}$ $\frac{7}{5}$ $\frac{9}{6}$ $\frac{16}{16}$ $\frac{20}{16}$

221 8. Add. Give your answers in simplest form.

$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$ $\frac{2}{3}+\frac{2}{3}$ $2\frac{1}{4}+2\frac{1}{4}+2\frac{1}{4}+2\frac{1}{4}$

76–79 9. Find the products.

$\frac{1}{2}$ of 12 $\frac{2}{3}$ of 15 $\frac{1}{4}\times 20$ $\frac{3}{4}\times 20$

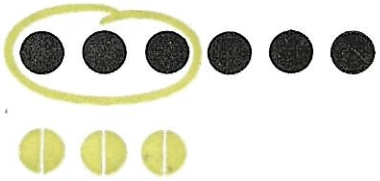
If you made mistakes in any row, turn to the page listed in front of the row. Study the explanation given there. Then work on the practice examples until you are sure you can do them correctly.

Thinking about Multiplying a Fraction and a Whole Number

ORAL REVIEW AND PREPARATION FOR MULTIPLYING FRACTIONS AND WHOLE NUMBERS

Find the products. You learned how to do examples like these on pages 76 to 78. Study those pages if you need help.

1. $\frac{1}{6} \times 12$ $\frac{5}{6} \times 12$ $\frac{1}{3} \times 6$ $\frac{2}{3} \times 6$ $\frac{1}{3} \times 9$ $\frac{2}{3} \times 9$
2. $\frac{1}{5} \times 10$ $\frac{2}{5} \times 10$ $\frac{4}{5} \times 10$ $\frac{1}{8} \times 24$ $\frac{3}{8} \times 24$ $\frac{7}{8} \times 24$
3. Does $4 + 4 + 4$ have the same meaning as 3×4?
4. Does $3 + 3 + 3 + 3$ have the same meaning as 4×3?
5. Does 3×4 have the same meaning as 4×3? Do they have the same answer?
6. Now think about $\frac{1}{2} \times 6$ and $6 \times \frac{1}{2}$. Which of the drawings at the right shows $\frac{1}{2} \times 6$? Which shows $6 \times \frac{1}{2}$? Is the answer 3 for each of them? Would you expect this to be true? Why?



7. How do you find $\frac{2}{3} \times 12$? How would you find $12 \times \frac{2}{3}$? Could you do other examples like this in the same way?

Find the products.

8. $12 \times \frac{1}{6}$ $12 \times \frac{5}{6}$ $6 \times \frac{1}{3}$ $6 \times \frac{2}{3}$ $9 \times \frac{1}{3}$ $9 \times \frac{2}{3}$
9. $10 \times \frac{1}{5}$ $10 \times \frac{2}{5}$ $10 \times \frac{4}{5}$ $24 \times \frac{1}{8}$ $24 \times \frac{3}{8}$ $24 \times \frac{7}{8}$
10. Compare row 1 with row 8 and row 2 with row 9. Should the answers be the same?

Tom said, "I have a way of multiplying a fraction and a whole number. Let me show you.

"To find $\frac{3}{4} \times 24$, I think: 4's in 24, _?_; $3 \times 6 =$ _?_.
"To find $24 \times \frac{3}{4}$, I think: 4's in 24, _?_; $6 \times 3 =$ _?_."

What do you think of Tom's way? Jane said, "Your way is fine for examples like that, but what do you do if the whole number and the denominator of the fraction will not divide evenly?"
Miss White said, "Let's work on that together."

Increasing a Recipe

A WHOLE NUMBER TIMES A FRACTION

Glenn brought this recipe to school for the cookbook. Tom and Sue worked with the other children to find the amounts needed to make 3 times the recipe.

Rolled Wafers

$\frac{1}{4}$ cup butter
$\frac{1}{2}$ cup powdered sugar
$\frac{7}{8}$ cup flour
$\frac{1}{4}$ cup milk
$\frac{1}{2}$ teaspoon vanilla

1. Sue said that to find 3 times $\frac{1}{4}$ cup of butter she would measure $\frac{1}{4}$ cup of butter 3 times. Sue added to find her answer. $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$. Sue said that 3 times the recipe would take $\frac{3}{4}$ cup of butter.

2. Glenn said that he found the answer by multiplying. 3 × 1 fourth = 3 fourths. Using figures, Glenn wrote $3 \times \frac{1}{4} = \frac{3}{4}$.

   Draw a picture to show that $3 \times \frac{1}{4} = \frac{3}{4}$.

3. Find 3 times $\frac{1}{2}$ cup of powdered sugar. Think: 3 × 1 half = 3 halves = $1\frac{1}{2}$. Write this in figures.

   Draw a picture to show that $3 \times \frac{1}{2} = 1\frac{1}{2}$.
   Three times the recipe would take $1\frac{1}{2}$ cups of powdered sugar.

4. To find 3 times $\frac{7}{8}$ cup of flour, Tom thought: 3 × 7 eighths = 21 eighths = $2\frac{5}{8}$. He wrote it in figures: $3 \times \frac{7}{8} = \frac{3 \times 7}{8} = \frac{21}{8} = 2\frac{5}{8}$

   Draw a picture to show that $3 \times \frac{7}{8} = 2\frac{5}{8}$.
   Three times the recipe would take $2\frac{5}{8}$ cups of flour.

5. Show how Tom found how much milk and how much vanilla he would need in 3 times the recipe. Then copy the new recipe on a sheet of paper beside the old recipe.

6. Can you give a rule for finding a whole number times a fraction?

7. The class decided to find 4 times the recipe for the Rolled Wafers. This is the way Glenn did his work. Finish each

example by giving the correct answer. Express all answers in simplest form.

$4 \times \frac{1}{4} = \frac{4 \times 1}{4} = \underline{?}$ $\quad$ $4 \times \frac{1}{4} = \frac{4 \times 1}{4} = \underline{?}$

$4 \times \frac{1}{2} = \frac{4 \times 1}{2} = \underline{?}$ $\quad$ $4 \times \frac{1}{2} = \frac{4 \times 1}{2} = \underline{?}$

$4 \times \frac{7}{8} = \frac{4 \times 7}{8} = \underline{?}$

Here is Glenn's rule for multiplying a fraction by a whole number. Is it like your rule?

To multiply a fraction by a whole number, multiply the numerator of the fraction by the whole number and divide the product by the denominator.

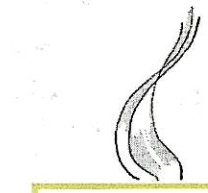
Use Glenn's rule to find the products. Express all answers in simplest form.

8.	$3 \times \frac{1}{2}$	$6 \times \frac{1}{2}$	$3 \times \frac{1}{3}$	$7 \times \frac{1}{3}$
9.	$5 \times \frac{1}{6}$	$6 \times \frac{3}{4}$	$2 \times \frac{2}{5}$	$3 \times \frac{3}{4}$
10.	$6 \times \frac{2}{3}$	$7 \times \frac{3}{8}$	$4 \times \frac{5}{6}$	$7 \times \frac{7}{8}$
11.	$10 \times \frac{7}{9}$	$12 \times \frac{3}{5}$	$9 \times \frac{7}{10}$	$14 \times \frac{2}{7}$
12.	$21 \times \frac{2}{3}$	$32 \times \frac{5}{6}$	$20 \times \frac{3}{8}$	$12 \times \frac{2}{5}$
13.	$27 \times \frac{5}{8}$	$20 \times \frac{3}{7}$	$16 \times \frac{2}{5}$	$69 \times \frac{1}{3}$

Jane said, "In some of those examples we could have divided both the numerator and the denominator by the same number before we multiplied. It might be easier to divide first when we can, because then we would have smaller numbers to work with."

14. Study the following examples. Then do the first two examples in row 12 in this way.

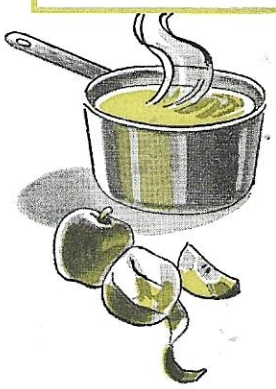
$6 \times \frac{2}{3} = \frac{\overset{2}{\cancel{6}} \times 2}{\underset{1}{\cancel{3}}} = \frac{4}{1} = 4$ $\quad$ $20 \times \frac{3}{8} = \frac{\overset{5}{\cancel{20}} \times 3}{\underset{2}{\cancel{8}}} = \frac{15}{2} = 7\frac{1}{2}$



Decreasing a Recipe

A FRACTION TIMES A WHOLE NUMBER

Spicy Applesauce
12 apples 2 cups water
1 cup sugar 16 whole cloves
Wash, peel, cut, and core apples. Cook slowly 30 minutes.



John wants to make this recipe for Spicy Applesauce, but he has only 9 apples. How can he tell what amounts to use of the other things?

John said, "I have to find out what part of 12 apples my 9 apples are. $\frac{9}{12} = \frac{3}{4}$. I'll make $\frac{3}{4}$ of the recipe. I'll multiply every amount by $\frac{3}{4}$."

Roy said, "The sugar is easy. To find $\frac{3}{4}$ of 1 cup, just fill the cup $\frac{3}{4}$ full of sugar. We could find $\frac{3}{4}$ of 2 cups of water by filling 2 cups each $\frac{3}{4}$ full."

John said, "That would really be finding $2 \times \frac{3}{4}$ instead of $\frac{3}{4} \times 2$, but it would be all right, because both of them make the same amount."

1. Why does $2 \times \frac{3}{4} = \frac{3}{4} \times 2$? Could you use the same rule for both?

$$\frac{3}{4} \times 2 = \frac{3 \times 2}{4} = \frac{6}{4} = 1\frac{2}{4} = 1\frac{1}{2} \quad \text{or} \quad \frac{3 \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}}} = \frac{3}{2} = 1\frac{1}{2}$$

2. How many cloves will John need?

3. Make one rule to use both for examples like $\frac{2}{3} \times 10$ and for examples like $10 \times \frac{2}{3}$. Use your rule to find the amounts needed for $\frac{2}{3}$ of the recipe.

To multiply a whole number by a fraction or a fraction by a whole number, multiply the whole number by the numerator of the fraction and divide the product by the denominator.

Find the products. Express all answers in simplest form.

4. $\frac{2}{3} \times 10$	$\frac{2}{5} \times 25$	$\frac{2}{5} \times 16$	$\frac{3}{4} \times 28$
5. $\frac{2}{7} \times 25$	$\frac{5}{6} \times 15$	$\frac{7}{8} \times 5$	$\frac{4}{5} \times 16$
6. $\frac{2}{3} \times 17$	$\frac{3}{8} \times 36$	$\frac{2}{9} \times 72$	$\frac{3}{4} \times 15$

Changing Mixed Numbers to Improper Fractions

This diagram shows that $2\frac{1}{3} = \frac{7}{3}$

$1 + 1 + \frac{1}{3} = 2\frac{1}{3}$

$\frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{7}{3}$

$2\frac{1}{3} = \frac{7}{3}$

1. Without the diagram, can you think of a way to show that $2\frac{1}{3}$ equals $\frac{7}{3}$? $1 = \frac{3}{3}$, so $2 = \frac{2 \times 3}{3} = \frac{6}{3}$; $2\frac{1}{3} = \frac{6}{3} + \frac{1}{3} = \frac{6+1}{3} = \frac{?}{?}$.
2. To change $1\frac{1}{4}$ to an improper fraction, think: $1 = \frac{4}{4}$, so $1\frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{4+1}{4} = \frac{?}{4}$.
3. To change $2\frac{3}{4}$ to an improper fraction, think: $1 = \frac{4}{4}$, so $2 = \frac{2 \times 4}{4} = \frac{8}{4}$; $\frac{8}{4} + \frac{3}{4} = \frac{?}{4}$; $2\frac{3}{4} = \frac{?}{?}$.
4. Tell how you think when finding the number to replace each of these question marks:

$1\frac{2}{3} = \frac{?}{3}$	$5\frac{3}{4} = \frac{?}{4}$	$3\frac{2}{5} = \frac{?}{5}$	$6\frac{3}{8} = \frac{?}{8}$
$1\frac{1}{2} = \frac{?}{2}$	$4\frac{5}{8} = \frac{?}{8}$	$2\frac{1}{6} = \frac{?}{6}$	$1\frac{4}{9} = \frac{?}{9}$

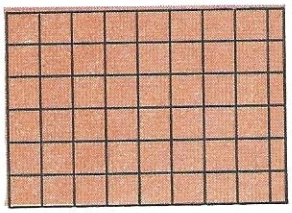
5. In exercise 4, how did you know what number to use for the denominator of each improper fraction? How did you find the numerator?

A quick way to change $3\frac{5}{8}$ to an improper fraction would be to think: In 3, there are 3×8 eighths, or 24 eighths. Then I have to add the other 5 eighths. $3 \times 8 = 24$, add 5, 29. $3\frac{5}{8} = \frac{29}{8}$. In the same way, $2\frac{1}{6} = \frac{2 \times 6 + 1}{6} = \frac{13}{6}$.

Change these mixed numbers to improper fractions.

6. $8\frac{1}{7}$	$2\frac{1}{8}$	$6\frac{4}{5}$	$9\frac{3}{8}$	$3\frac{3}{5}$	$2\frac{2}{3}$	$1\frac{5}{6}$
7. $10\frac{1}{8}$	$5\frac{7}{10}$	$2\frac{3}{8}$	$12\frac{3}{4}$	$5\frac{2}{9}$	$6\frac{7}{8}$	$7\frac{1}{5}$
8. $6\frac{2}{3}$	$5\frac{3}{8}$	$4\frac{9}{10}$	$3\frac{5}{6}$	$2\frac{7}{10}$	$1\frac{3}{4}$	$9\frac{1}{10}$

A Mixed Number Times a Whole Number



Wayne is fencing off a pen for his dog. The pen is $8\frac{1}{2}$ feet long and 6 feet wide. What is the area of the pen?

In the drawing at the left, each whole square stands for 1 square foot. What does half a square stand for? How many whole squares are there? How many half squares? How many whole squares would these half squares make? Add the whole squares and the half squares in the drawing to find the area of the pen.

$$\begin{array}{r} 6 \\ 8\frac{1}{2} \\ \hline 3 \quad (\frac{1}{2} \times 6) \\ 48 \quad (8 \times 6) \\ \hline 51 \end{array}$$

Wayne found the area by multiplying $8\frac{1}{2} \times 6$. First he multiplied $\frac{1}{2} \times 6 = 3$. Then he multiplied $8 \times 6 = 48$. Why did he write the 8 in 48 directly under the 3 in his example? He added the two products. $3 + 48 = 51$. Compare Wayne's method with what you did when you used the drawing to find the answer.

Alice said she could find the answer in another way. She knew how to find a fraction times a whole number, so she changed $8\frac{1}{2}$ to an improper fraction. Then she multiplied by that fraction. $\frac{17}{2} \times 6 = \frac{102}{2} = 51$

In her example, could Alice have divided before she multiplied?

The area of the pen is 51 square feet.

Multiply. Use the method you like better. Express your answers in simplest form.

1. $1\frac{3}{8} \times 7$	$8\frac{1}{2} \times 5$	$3\frac{3}{4} \times 5$	$5\frac{3}{5} \times 4$
2. $1\frac{3}{4} \times 8$	$5\frac{5}{8} \times 5$	$9\frac{1}{4} \times 6$	$7\frac{2}{3} \times 8$
3. $8\frac{1}{4} \times 8$	$8\frac{3}{5} \times 5$	$17\frac{1}{6} \times 3$	$12\frac{2}{3} \times 5$
4. $9\frac{1}{6} \times 8$	$8\frac{2}{5} \times 4$	$6\frac{2}{3} \times 9$	$8\frac{1}{3} \times 3$

5. Study the two ways of multiplying $3\frac{3}{4} \times 11$ shown at the top of the next page.

(a) $3\frac{3}{4} \times 11 = \frac{15}{4} \times 11 = \frac{15 \times 11}{4} = \frac{165}{4} = 41\frac{1}{4}$

Could you divide before multiplying in this example? Can you multiply 15×11 mentally? Can you divide $165 \div 4$ mentally?

(b) 11

$3\frac{3}{4}$

$8\frac{1}{4}$

33

$41\frac{1}{4}$

How do we get the $8\frac{1}{4}$? The 33?

Most people think that this way is easier than (a) when the numbers to multiply are large.

6. Mary earns \$.75 an hour helping to take care of children. How much does she earn when she works $2\frac{1}{2}$ hours?

Explain the two ways of finding the answer.

(a) $2\frac{1}{2} \times \$.75 = \frac{5}{2} \times \$.75 = \frac{5 \times \$.75}{2} = \frac{\$3.75}{2} = \$1.87\frac{1}{2}$, or \$1.88

(b) \$ .75

$2\frac{1}{2}$

$37\frac{1}{2}$

1 50

$\$1.87\frac{1}{2}$, or \$1.88

Why should $\$1.87\frac{1}{2}$ be changed to \$1.88?

What is the answer to the question the problem asks?

Copy and multiply. Express your answers in simplest form.

7. $15 \times 8\frac{1}{5}$ $\quad 24 \times 7\frac{3}{4}$ $\quad 36 \times 6\frac{2}{3}$ $\quad 28 \times 5\frac{5}{7}$ $\quad 32 \times 4\frac{3}{4}$ $\quad 20 \times 6\frac{2}{5}$

8. $30 \times 5\frac{2}{3}$ $\quad 16 \times 9\frac{1}{2}$ $\quad 25 \times 4\frac{4}{5}$ $\quad 60 \times 4\frac{5}{6}$ $\quad 36 \times 6\frac{5}{12}$ $\quad 120 \times 2\frac{11}{12}$

9. $\$.24 \times 4\frac{1}{2}$ $\quad \$.48 \times 3\frac{1}{3}$ $\quad \$.16 \times 6\frac{3}{4}$ $\quad \$.36 \times 5\frac{5}{9}$ $\quad \$.80 \times 7\frac{1}{9}$ $\quad \$.40 \times 6\frac{1}{6}$

10. Work each of the following division examples to see if you get the same answer as the one in the book. Then check each answer by multiplying the divisor and the quotient.

$2\overline{)9}$ $\quad 4\frac{1}{2}$

$21\overline{)133}$ $\quad 6\frac{1}{3}$

$35\overline{)294}$ $\quad 8\frac{2}{5}$

$24\overline{)876}$ $\quad 36\frac{1}{2}$

A Whole Number Times a Mixed Number

Mrs. Nelson made 4 fruit cakes. If each cake weighed $2\frac{1}{4}$ pounds, how much did the cakes weigh all together?

$2\frac{1}{4}$
4
$1\ (4 \times \frac{1}{4})$
$8\ (4 \times 2)$
9

John said, "I can find the answer by adding.

$$2\tfrac{1}{4} + 2\tfrac{1}{4} + 2\tfrac{1}{4} + 2\tfrac{1}{4} = 8\tfrac{4}{4} = 9."$$

Sue said, "I can find the answer by multiplying.
First I will multiply $4 \times \frac{1}{4}$. $4 \times \frac{1}{4} = \frac{4}{4} = 1$.
Next I will multiply 4×2. $4 \times 2 = 8$.
Then I will add the two products together. $1 + 8 = 9$."

Bob said, "I can find the answer by multiplying in a different way. I know how to multiply a fraction by a whole number, so I will change $2\frac{1}{4}$ to an improper fraction. $2\frac{1}{4} = \frac{8+1}{4} = \frac{9}{4}$. Then I will multiply by 4. $4 \times 2\frac{1}{4} = 4 \times \frac{9}{4} = \frac{36}{4} = 9$. The total weight of the fruit cakes was 9 pounds."

Which way do you think is the best way to find the answer?

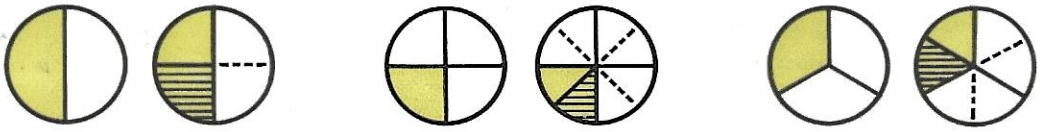
Bob's way is sometimes the best way to do examples like this, and we shall need to use it in other kinds of examples later, so we should think about Bob's way.

Notice that $4 \times 2\frac{1}{4} = 4 \times \frac{9}{4} = \frac{36}{4} = 9$. Could Bob have divided before he multiplied in this example? It makes the numbers smaller and easier to work with if we divide before we multiply when possible.

Copy and use Bob's way to find the products. Express your answers in simplest form.

1.	$4 \times 6\frac{1}{4}$	$8 \times 5\frac{1}{2}$	$6 \times 4\frac{1}{3}$	$9 \times 7\frac{1}{8}$
2.	$5 \times 8\frac{1}{2}$	$9 \times 6\frac{1}{3}$	$8 \times 8\frac{1}{6}$	$3 \times 4\frac{1}{8}$
3.	$4 \times 7\frac{1}{6}$	$6 \times 8\frac{4}{5}$	$4 \times 3\frac{5}{6}$	$15 \times 2\frac{2}{3}$
4.	$5 \times 1\frac{3}{10}$	$3 \times 1\frac{1}{12}$	$4 \times 2\frac{3}{8}$	$6 \times 2\frac{1}{10}$

5. John picked 7 boxes of peaches. Each box holds $1\frac{1}{4}$ bushels. How many bushels did John pick?



$\frac{1}{2}$ $\frac{1}{2}$ of $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{2}$ of $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ of $\frac{1}{3}$

Finding a Part of a Part

Make drawings similar to those above, or use fraction cutouts if you need them to answer questions 1–5 below.

1. Joan wishes to make half a recipe for a layer cake. The whole recipe calls for $\frac{1}{2}$ cup shortening. How much shortening will Joan need? $\frac{1}{2}$ of $\frac{1}{2}$ = _?_

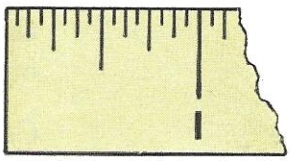
2. Mrs. Jones cut a cake into 4 equal parts. Each part is _?_ of the cake. She told Bill and Jim that they could share one of the fourths. How much cake will each boy receive? $\frac{1}{2}$ of $\frac{1}{4}$ = _?_

3. Does $\frac{1}{2}$ of $\frac{1}{4}$ make the same amount as $\frac{1}{4}$ of $\frac{1}{2}$?

4. What is $\frac{1}{3}$ of $\frac{3}{4}$? What is $\frac{1}{4}$ of $\frac{1}{2}$? What is $\frac{1}{3}$ of $\frac{1}{2}$?

5. What is $\frac{1}{2}$ of $\frac{1}{3}$? What is $\frac{1}{2}$ of $\frac{2}{3}$? What is $\frac{1}{2}$ of $\frac{3}{4}$?

6. Look at the drawing of part of a ruler. How much is $\frac{1}{2}$ of $\frac{1}{2}$ inch? $\frac{1}{4}$ of $\frac{1}{2}$ inch? $\frac{1}{2}$ of $\frac{1}{4}$ inch?



7. You know that these three examples mean the same thing:

$\frac{1}{4}$ of $\frac{1}{2}$ $\quad$ $\frac{1}{4}$ times $\frac{1}{2}$ $\quad$ $\frac{1}{4} \times \frac{1}{2}$

Use the third way of writing these to show why $\frac{1}{4}$ of $\frac{1}{2}$ makes the same amount as $\frac{1}{2}$ of $\frac{1}{4}$.

Copy these examples and write the answers. Use drawings or fraction cutouts if you need help.

8. $\frac{1}{3} \times \frac{1}{2}$ $\quad$ $\frac{1}{3} \times \frac{3}{4}$ $\quad$ $\frac{1}{4} \times \frac{1}{2}$ $\quad$ $\frac{1}{2} \times \frac{3}{4}$ $\quad$ $\frac{1}{2} \times \frac{1}{3}$ $\quad$ $\frac{1}{2} \times \frac{2}{3}$

9. In the examples you have just finished, is each product smaller than either of the two fractions that you multiplied? Why would you expect this to be true?

Multiplying a Fraction by a Fraction

DIVIDING BEFORE MULTIPLYING

1. What is the answer called in a multiplication example?

2. Study the examples below. In each product, how was the numerator found? How was the denominator found?

$$\frac{1}{3} \times \frac{1}{4} = \frac{1 \times 1}{3 \times 4} = \frac{1}{12} \qquad \frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7} = \frac{15}{56}$$

> To multiply fractions, multiply the numerators to get the numerator of the product. Multiply the denominators to get the denominator of the product.

3. Which of these two ways to multiply $\frac{2}{3} \times \frac{1}{4}$ do you like better?

$$\frac{2}{3} \times \frac{1}{4} = \frac{2 \times 1}{3 \times 4} = \frac{2}{12} = \frac{1}{6} \qquad \frac{2}{3} \times \frac{1}{4} = \frac{\overset{1}{\cancel{2}} \times 1}{3 \times \underset{2}{\cancel{4}}} = \frac{1}{6}$$

When do you divide a numerator and a denominator by the same number in the first way? In the second way? Do you see that when you divide before you multiply, the product comes out in lowest terms?

Copy and multiply. Express all products in lowest terms.

4. $\frac{1}{2} \times \frac{1}{3}$ $\quad \frac{1}{3} \times \frac{1}{4}$ $\quad \frac{1}{4} \times \frac{1}{3}$ $\quad \frac{1}{2} \times \frac{3}{5}$

5. $\frac{1}{3} \times \frac{4}{5}$ $\quad \frac{1}{4} \times \frac{3}{5}$ $\quad \frac{1}{5} \times \frac{2}{3}$ $\quad \frac{1}{5} \times \frac{1}{4}$

6. $\frac{1}{2} \times \frac{1}{4}$ $\quad \frac{1}{2} \times \frac{1}{5}$ $\quad \frac{1}{3} \times \frac{1}{2}$ $\quad \frac{1}{3} \times \frac{1}{5}$

7. $\frac{1}{4} \times \frac{1}{2}$ $\quad \frac{1}{4} \times \frac{1}{5}$ $\quad \frac{1}{3} \times \frac{1}{3}$ $\quad \frac{1}{4} \times \frac{1}{4}$

8. $\frac{1}{5} \times \frac{3}{4}$ $\quad \frac{1}{2} \times \frac{3}{4}$ $\quad \frac{1}{3} \times \frac{2}{5}$ $\quad \frac{1}{4} \times \frac{3}{4}$

9. $\frac{3}{4} \times \frac{1}{2}$ $\quad \frac{2}{3} \times \frac{1}{4}$ $\quad \frac{2}{5} \times \frac{1}{2}$ $\quad \frac{3}{4} \times \frac{2}{3}$

10. $\frac{2}{3} \times \frac{1}{2}$ $\quad \frac{2}{3} \times \frac{1}{3}$ $\quad \frac{3}{4} \times \frac{1}{4}$ $\quad \frac{2}{3} \times \frac{1}{5}$

11. $\frac{3}{5} \times \frac{1}{4}$ $\frac{2}{3} \times \frac{4}{5}$ $\frac{4}{5} \times \frac{2}{3}$ $\frac{3}{5} \times \frac{3}{5}$ $\frac{3}{4} \times \frac{4}{5}$

12. $\frac{3}{5} \times \frac{1}{2}$ $\frac{5}{6} \times \frac{1}{2}$ $\frac{4}{5} \times \frac{1}{2}$ $\frac{5}{6} \times \frac{3}{4}$ $\frac{2}{3} \times \frac{3}{5}$

Shown below are four correct ways of writing the same example.

$\frac{2}{3} \times \frac{5}{6} = \frac{2 \times 5}{3 \times 6} = \frac{10}{18}, \quad \frac{10 \div 2}{18 \div 2} = \frac{5}{9}$

$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{1}{3} \times \frac{5}{3} = \frac{5}{9}$

$\frac{2}{3} \times \frac{5}{6} = \frac{\overset{1}{\cancel{2}} \times 5}{3 \times \underset{3}{\cancel{6}}} = \frac{5}{9}$

$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{3}{\cancel{6}}} = \frac{1 \times 5}{3 \times 3} = \frac{5}{9}$

In all four ways, were one numerator and one denominator divided by the same number? What number was it? Why was 2 chosen to divide by?

Why is it a good idea to cross out the numbers that have been divided?

The fundamental principle of fractions states that you can divide the numerator and denominator of a fraction by the same number without changing the value of the fraction. Was this principle used here? Explain.

Copy and multiply. Use the method you like best. Express all products in lowest terms.

13. $\frac{1}{2} \times \frac{4}{5}$ $\frac{2}{5} \times \frac{3}{4}$ $\frac{3}{4} \times \frac{1}{6}$ $\frac{9}{10} \times \frac{1}{3}$ $\frac{1}{2} \times \frac{6}{7}$

14. $\frac{3}{4} \times \frac{1}{12}$ $\frac{5}{8} \times \frac{2}{3}$ $\frac{3}{5} \times \frac{2}{3}$ $\frac{5}{6} \times \frac{3}{4}$ $\frac{1}{4} \times \frac{8}{9}$

15. $\frac{4}{5} \times \frac{3}{16}$ $\frac{4}{7} \times \frac{14}{15}$ $\frac{5}{8} \times \frac{3}{5}$ $\frac{3}{8} \times \frac{5}{6}$ $\frac{4}{5} \times \frac{1}{8}$

16. $\frac{2}{3} \times \frac{6}{7}$ $\frac{3}{10} \times \frac{5}{8}$ $\frac{3}{16} \times \frac{4}{5}$ $\frac{3}{5} \times \frac{1}{15}$ $\frac{4}{7} \times \frac{1}{4}$

17. $\frac{1}{2} \times \frac{8}{9}$ $\frac{2}{3} \times \frac{1}{6}$ $\frac{3}{4} \times \frac{1}{9}$ $\frac{2}{3} \times \frac{5}{8}$ $\frac{6}{7} \times \frac{1}{3}$

18. $\frac{1}{3} \times \frac{3}{4}$ $\frac{2}{3} \times \frac{1}{8}$ $\frac{1}{4} \times \frac{4}{5}$ $\frac{4}{5} \times \frac{3}{8}$ $\frac{1}{3} \times \frac{9}{10}$

19. $\frac{3}{4} \times \frac{4}{5}$ $\frac{2}{3} \times \frac{3}{5}$ $\frac{4}{5} \times \frac{7}{8}$ $\frac{5}{8} \times \frac{3}{10}$ $\frac{5}{6} \times \frac{2}{7}$

20. $\frac{3}{4} \times \frac{1}{3}$ $\frac{1}{5} \times \frac{5}{8}$ $\frac{7}{12} \times \frac{3}{4}$ $\frac{3}{5} \times \frac{5}{8}$ $\frac{1}{3} \times \frac{3}{8}$

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–6)

1. Round these numbers to the nearest thousand.

   5478 6742 2901 7680 8299

2. Write in figures: Three million, two hundred twenty thousand, three hundred five.

3. Write the Arabic numbers for: XXXVIII XCV DCIX

4. Find the perimeter of a rectangle 46 ft. by 65 ft.

5. Find the area of a rectangle 36 yd. by 54 yd.

6. 84 in. = _?_ ft. 48 oz. = _?_ lb. 6 sq. yd. = _?_ sq. ft.

Copy. Add. Check your work.

7.

804	$3.74	583	497.80	$14.89
762	6.33	2,797	301.64	.87
41	5.64	944	721.93	3.00
59	2.05	6,977	546.70	30.05

8.

$\frac{1}{8}$	$\frac{1}{6}$	$\frac{3}{4}$	$12\frac{1}{3}$	$7\frac{5}{6}$	$9\frac{1}{3}$
$\frac{3}{8}$	$\frac{3}{6}$	$\frac{1}{2}$	$6\frac{5}{12}$	$9\frac{2}{3}$	$5\frac{3}{5}$

Copy. Subtract. Check your work.

9.

926	$9.21	7,048	266.78	$47.50
543	5.16	2,989	129.68	15.95

10.

$\frac{5}{6}$	$\frac{8}{9}$	$\frac{2}{3}$	9	$15\frac{5}{8}$	$8\frac{1}{4}$
$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{5}$	$3\frac{1}{4}$	$4\frac{2}{3}$

Copy. Multiply. Check your work.

11.

852	409	179	$7.30	$18.55
60	58	209	1 35	42

Copy. Divide. Check your work.

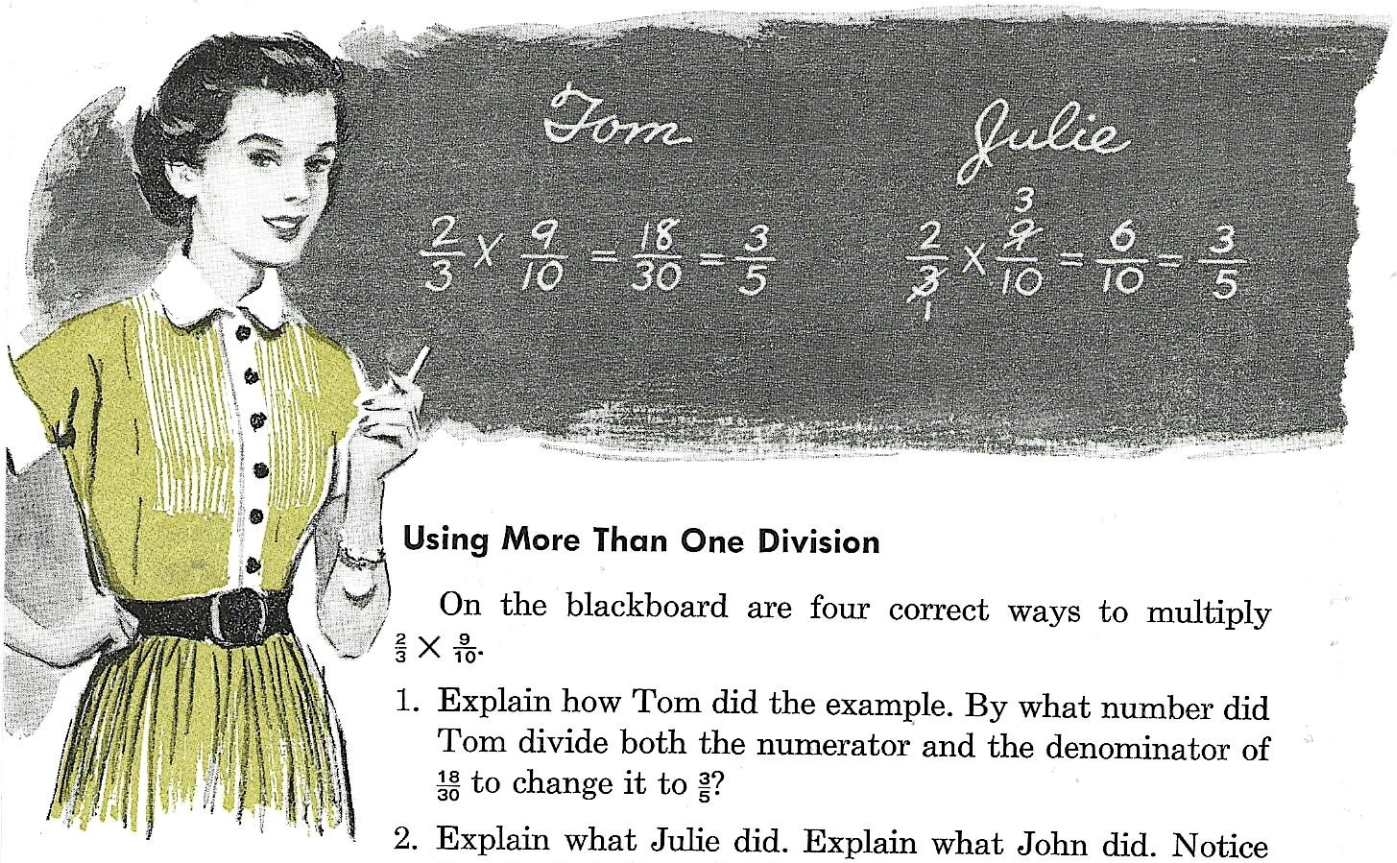
12. $23\overline{)599}$ $31\overline{)2{,}170}$ $63\overline{)21{,}547}$ $68\overline{)\$33.12}$ $215\overline{)\$53.95}$

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–6)

Read each problem carefully. Decide whether to add, subtract, multiply, or divide. Then find the answer.

1. The Jefferson School playground is 264 feet by 165 feet. What is the perimeter of this playground? What is its area?
2. Charles bought a belt for $1.25, a pair of moccasins for $7.95, and a shirt for $4.50. How much in all did he pay for these?
3. Last year the camp museum had 123 rock specimens. This year they have 152. How many more rock specimens does the camp have this year than last year?
4. A freight train traveled 1,496 miles in 44 hours. How many miles an hour did it travel?
5. The boys' relay team ran the first lap of a race in 2 minutes 15 seconds and the second lap in 1 minute 55 seconds. How long did it take the relay team to run both laps of the race?
6. To help in timing races, Bob cut 6 string cords each 12 feet 3 inches long for the finish line. How much string cord did he use in all?
7. In the high jump, Bill reached 4 feet 5 inches and Sam reached 3 feet 11 inches. How much higher did Bill jump than Sam?
8. June had 2 pounds 8 ounces of candy which she wished to pack in 5 bags of equal weight. How much candy should she pack in each bag?
9. Mrs. Martin bought three boxes of tomatoes. They weighed $5\frac{1}{2}$ pounds, $6\frac{1}{4}$ pounds, and $5\frac{3}{8}$ pounds. How much did all three boxes weigh?
10. At the beginning of the week, Mrs. Park had $2\frac{1}{4}$ pounds butter. She has $1\frac{1}{8}$ pounds left. How much butter did she use?
11. In a recent airplane race a winner flew 507.255 miles an hour, and in another race a winner flew 460.423 miles an hour. How much faster did one winner fly than the other?
12. A commercial airplane flew 196.6 miles in the first hour, 203.7 miles in the second, 193.8 miles in the third, and 211.3 miles in the fourth. How far did it travel in the four hours?

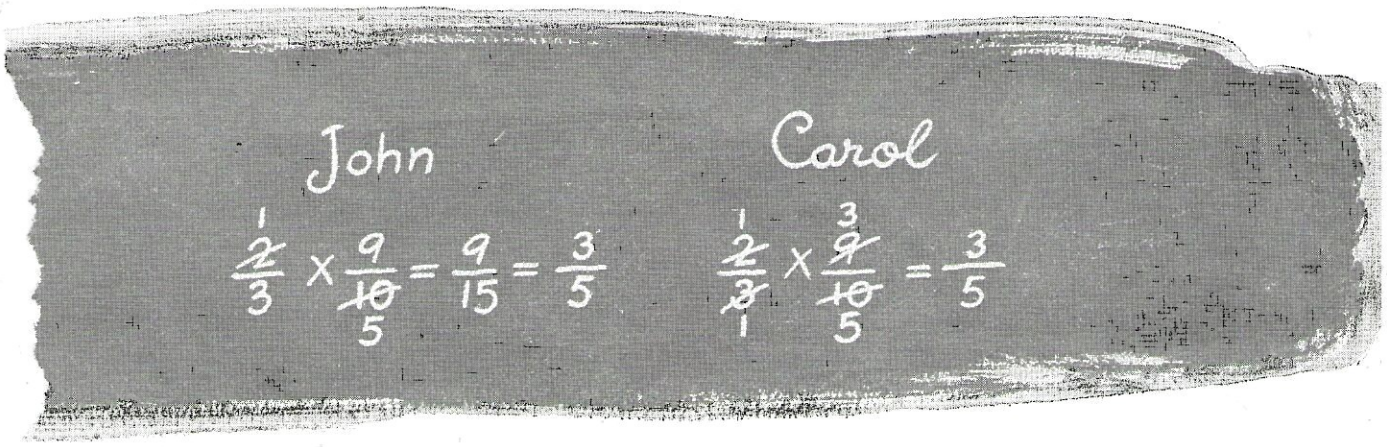


Using More Than One Division

On the blackboard are four correct ways to multiply $\frac{2}{3} \times \frac{9}{10}$.

1. Explain how Tom did the example. By what number did Tom divide both the numerator and the denominator of $\frac{18}{30}$ to change it to $\frac{3}{5}$?
2. Explain what Julie did. Explain what John did. Notice that both Julie and John could change their answers to lower terms.
3. Carol combined Julie's work and John's work. By what two numbers did Carol divide before she multiplied? Did Carol's answer come out in lowest terms?
4. When you use division more than once in an example, does it matter which division you use first? Does dividing by 2, then by 3, give the same result as dividing by 3, then by 2? Do both of them amount to dividing by 6?

If the fractions in the example are not in lowest terms, or if you do not use the largest possible number when you divide a numerator and a denominator, you may have several cancellations in one example. When this happens, you must record each of your divisions carefully.



Study the following examples. Remember that the divisions would not have to be done in this order.

(a) $\frac{3}{4} \times \frac{8}{12}$

Divide 3 and 12 by 3: $\frac{\cancel{3}^{1}}{4} \times \frac{8}{\cancel{12}_{4}}$

Divide 4 and 8 by 4: $\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{12}_{4}}$

Divide 2 and 4 by 2: $\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{1}}{\cancel{12}_{\cancel{4}^{2}}} = \frac{1}{2}$

(b) $\frac{5}{6} \times \frac{18}{25}$

Divide 5 and 25 by 5: $\frac{\cancel{5}^{1}}{6} \times \frac{18}{\cancel{25}_{5}}$

Divide 6 and 18 by 3: $\frac{\cancel{5}^{1}}{\cancel{6}_{2}} \times \frac{\cancel{18}^{6}}{\cancel{25}_{5}}$

Divide 2 and 6 by 2: $\frac{\cancel{5}^{1}}{\cancel{6}_{\cancel{2}^{1}}} \times \frac{\cancel{18}^{3}}{\cancel{25}_{5}} = \frac{3}{5}$

Multiply. Divide before multiplying wherever possible.

5.	$\frac{2}{3} \times \frac{3}{4}$	$\frac{5}{12} \times \frac{3}{10}$	$\frac{2}{7} \times \frac{7}{10}$	$\frac{2}{3} \times \frac{3}{6}$	$\frac{8}{9} \times \frac{9}{10}$
6.	$\frac{3}{10} \times \frac{2}{3}$	$\frac{2}{3} \times \frac{6}{8}$	$\frac{3}{12} \times \frac{3}{15}$	$\frac{3}{10} \times \frac{5}{15}$	$\frac{3}{4} \times \frac{4}{6}$
7.	$\frac{3}{8} \times \frac{2}{3}$	$\frac{5}{8} \times \frac{8}{15}$	$\frac{2}{3} \times \frac{9}{10}$	$\frac{4}{5} \times \frac{5}{10}$	$\frac{3}{5} \times \frac{5}{6}$
8.	$\frac{2}{14} \times \frac{7}{10}$	$\frac{5}{15} \times \frac{3}{10}$	$\frac{3}{12} \times \frac{4}{16}$	$\frac{2}{3} \times \frac{15}{16}$	$\frac{3}{10} \times \frac{5}{27}$
9.	$\frac{3}{10} \times \frac{5}{6}$	$\frac{2}{3} \times \frac{21}{24}$	$\frac{3}{10} \times \frac{5}{21}$	$\frac{7}{8} \times \frac{16}{21}$	$\frac{2}{3} \times \frac{12}{16}$
10.	$\frac{5}{8} \times \frac{2}{5}$	$\frac{3}{10} \times \frac{5}{9}$	$\frac{7}{12} \times \frac{2}{10}$	$\frac{8}{9} \times \frac{3}{4}$	$\frac{3}{10} \times \frac{5}{18}$
11.	$\frac{7}{10} \times \frac{5}{14}$	$\frac{2}{3} \times \frac{2}{3}$	$\frac{5}{21} \times \frac{7}{10}$	$\frac{5}{8} \times \frac{5}{8}$	$\frac{2}{3} \times \frac{24}{28}$
12.	$\frac{2}{12} \times \frac{3}{15}$	$\frac{7}{10} \times \frac{10}{14}$	$\frac{5}{6} \times \frac{5}{6}$	$\frac{7}{10} \times \frac{2}{14}$	$\frac{7}{8} \times \frac{7}{8}$

Joan's Cake Recipe Is Changed

PRODUCT OF A MIXED NUMBER AND A FRACTION

Jiffy Layer Cake	
$2\frac{2}{3}$ cups flour	$\frac{3}{4}$ cup shortening
$4\frac{1}{2}$ tsp. baking powder	1 cup milk
$\frac{1}{2}$ tsp. salt	1 tsp. vanilla
$1\frac{1}{2}$ cups sugar	4 eggs

This is Joan's favorite recipe for cake.

1. Joan wants to make only $\frac{3}{4}$ of the recipe. How much salt, milk, and vanilla will she need? How many eggs?

2. To tell how much flour she needs, Joan must find $\frac{3}{4}$ of $2\frac{2}{3}$ cups of flour.

   Joan knows how to multiply two fractions, so she changed $2\frac{2}{3}$ to an improper fraction. Then she multiplied by $\frac{3}{4}$. She made her work easier by dividing before multiplying.

   Explain her work. $2\frac{2}{3} = \frac{8}{3}$. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{1}{\cancel{3}}} = \frac{2}{1} = 2$.

   Joan will need 2 cups of flour.

3. To tell how much baking powder she needs, Joan must find $\frac{3}{4}$ of $4\frac{1}{2}$ teaspoons.

   Explain her work. $\frac{3}{4} \times 4\frac{1}{2} = \frac{3}{4} \times \frac{9}{2} = \frac{27}{8} = 3\frac{3}{8}$. Why did Joan not divide in the second step of this example? Joan will need $3\frac{3}{8}$ teaspoons of baking powder.

4. Make a rule for multiplying a mixed number by a fraction. Use your rule to find how much sugar Joan will need for $\frac{3}{4}$ of the recipe.

5. What would you do to multiply a mixed number and a fraction if the mixed number were the multiplier? Would you expect $8\frac{1}{3} \times \frac{3}{4}$ to have the same answer as $\frac{3}{4} \times 8\frac{1}{3}$? Try it and see.

6. Show that the product of $3\frac{1}{2} \times \frac{3}{8}$ is the same as the product of $\frac{3}{8} \times 3\frac{1}{2}$.

To multiply a mixed number and a fraction, change the mixed number to an improper fraction. Then multiply the two fractions.

Copy and multiply. Check by changing the order of the numbers and multiplying again.

7.	$4\frac{2}{3} \times \frac{3}{8}$	$2\frac{2}{3} \times \frac{1}{2}$	$1\frac{1}{3} \times \frac{1}{4}$	$2\frac{1}{2} \times \frac{1}{2}$
8.	$2\frac{1}{2} \times \frac{7}{8}$	$9\frac{1}{2} \times \frac{2}{3}$	$7\frac{1}{2} \times \frac{7}{8}$	$2\frac{2}{3} \times \frac{5}{8}$
9.	$\frac{3}{4} \times 3\frac{1}{3}$	$\frac{5}{6} \times 1\frac{1}{2}$	$\frac{2}{3} \times 5\frac{1}{4}$	$\frac{3}{5} \times 2\frac{2}{9}$
10.	$\frac{6}{7} \times 4\frac{2}{3}$	$\frac{1}{3} \times 3\frac{3}{5}$	$\frac{8}{9} \times 1\frac{1}{8}$	$\frac{5}{6} \times 4\frac{1}{2}$

Practice in Multiplying a Fraction and a Whole Number

WRITING A WHOLE NUMBER AS A FRACTION

You know that a fraction may be thought of as a division. $\frac{3}{4}$ may mean $3 \div 4$. $\frac{4}{3}$ may mean $4 \div 3$. $\frac{4}{1}$ may mean $4 \div 1$.

1. When you divide any number by 1, the answer is _?_.
2. Write the fraction that will mean each of the following numbers divided by 1: 3 7 13 28 100 1492
3. Since the value of a number is not changed when the number is divided by 1, any whole number may be written as a fraction with a denominator of 1. This gives us another way to write the multiplication of a fraction and a whole number. Compare the following two ways to do these examples.

$2 \times \frac{1}{6} = \frac{\overset{1}{\cancel{2}} \times 1}{\underset{3}{\cancel{6}}} = \frac{1}{3}$ $\qquad$ $2 \times \frac{1}{6} = \frac{2}{1} \times \frac{1}{6} = \frac{\overset{1}{\cancel{2}} \times 1}{1 \times \underset{3}{\cancel{6}}} = \frac{1}{3}$

$\frac{3}{4} \times 12 = \frac{3 \times \overset{3}{\cancel{12}}}{\underset{1}{\cancel{4}}} = \frac{9}{1} = 9$ $\qquad$ $\frac{3}{4} \times 12 = \frac{3}{4} \times \frac{12}{1} = \frac{3 \times \overset{3}{\cancel{12}}}{\underset{1}{\cancel{4}} \times 1} = \frac{9}{1} = 9$

Copy and multiply. Use any correct method.

4.	$5 \times \frac{1}{10}$	$8 \times \frac{3}{4}$	$12 \times \frac{5}{6}$	$32 \times \frac{3}{8}$
5.	$36 \times \frac{7}{9}$	$3 \times \frac{5}{12}$	$12 \times \frac{3}{16}$	$22 \times \frac{3}{4}$
6.	$\frac{2}{3} \times 24$	$\frac{3}{8} \times 64$	$\frac{5}{6} \times 42$	$\frac{7}{8} \times 16$
7.	$\frac{5}{16} \times 8$	$\frac{5}{9} \times 45$	$\frac{7}{12} \times 4$	$\frac{7}{16} \times 12$

Using Fractions with Measures

FRACTION TIMES A WHOLE NUMBER

1. Robert wants to buy $\frac{3}{4}$ pound of nails. How many ounces should he get?
   One pound = _?_ ounces.
   $\frac{3}{4}$ of a pound = $\frac{3}{4}$ of 16 ounces. $\frac{3}{4} \times \frac{16}{1} = \frac{3}{\cancel{4}_{1}} \times \frac{\overset{4}{\cancel{16}}}{1} =$ _?_
   Robert should get _?_ ounces of nails.

2. How many inches are there in $\frac{3}{4}$ of a yard?
   A yard = _?_ inches.
   $\frac{3}{4}$ of a yard = $\frac{3}{4} \times 36$ inches = _?_ inches.

3. How many feet are in $\frac{1}{2}$ of a yard?
   A yard = 3 feet.
   $\frac{1}{2}$ of a yard = $\frac{1}{2} \times 3$ feet = _?_ feet.

Copy and fill in the blanks.

4. 1 yd. = _?_ in.
   $\frac{1}{2}$ yd. = _?_ in.
   $\frac{1}{4}$ yd. = _?_ in.
   $\frac{1}{3}$ yd. = _?_ in.
   $\frac{2}{3}$ yd. = _?_ in.
   $\frac{3}{8}$ yd. = _?_ in.
   $\frac{7}{8}$ yd. = _?_ in.

5. 1 lb. = _?_ oz.
   $\frac{1}{2}$ lb. = _?_ oz.
   $\frac{1}{4}$ lb. = _?_ oz.
   $\frac{1}{3}$ lb. = _?_ oz.
   $\frac{2}{3}$ lb. = _?_ oz.
   $\frac{1}{8}$ lb. = _?_ oz.
   $\frac{3}{8}$ lb. = _?_ oz.

6. 1 gal. = _?_ qt.
   $\frac{1}{2}$ gal. = _?_ qt.
   $\frac{1}{4}$ gal. = _?_ qt.
   $\frac{3}{4}$ gal. = _?_ qt.
   $\frac{1}{5}$ gal. = _?_ qt.

7. 1 yr. = _?_ mo.
   $\frac{1}{4}$ yr. = _?_ mo.
   $\frac{1}{2}$ yr. = _?_ mo.
   $\frac{1}{3}$ yr. = _?_ mo.
   $\frac{2}{3}$ yr. = _?_ mo.

8. 1 hr. = _?_ min.
   $\frac{1}{2}$ hr. = _?_ min.
   $\frac{1}{4}$ hr. = _?_ min.
   $\frac{1}{3}$ hr. = _?_ min.
   $\frac{2}{3}$ hr. = _?_ min.
   $\frac{3}{4}$ hr. = _?_ min.

9. 1 min. = _?_ sec.
   $\frac{1}{2}$ min. = _?_ sec.
   $\frac{1}{4}$ min. = _?_ sec.
   $\frac{3}{4}$ min. = _?_ sec.
   $\frac{1}{3}$ min. = _?_ sec.
   $\frac{2}{3}$ min. = _?_ sec.

Multiplying Three Fractions

Sometimes you can save time in solving a problem by multiplying 3 fractions at once. Use the same method as for 2 fractions. Multiply the numerators for a new numerator, the denominators for a new denominator. You may divide before multiplying or not, as you wish. Remember that in every division one numerator and one denominator must be divided by the same number.

Study the following examples.

(a) $\frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = \frac{2 \times 3 \times 7}{3 \times 4 \times 8} = \frac{42}{96} = \frac{7}{16}$ or $\frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{2}} \times \frac{7}{8} = \frac{7}{16}$

(b) $\frac{5}{6} \times \frac{3}{10} \times \frac{2}{9} = \frac{5 \times 3 \times 2}{6 \times 10 \times 9} = \frac{30}{540} = \frac{1}{18}$ or $\frac{\cancel{5}^{1}}{\cancel{6}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{10}_{2}} \times \frac{\cancel{2}^{1}}{\cancel{9}_{3}} = \frac{1}{18}$

1. In examples like these, there is often more than one correct way to divide before multiplying. Can you find another way for example (b) above?

Multiply. Express your answer in lowest terms.

2. $\frac{3}{4} \times \frac{5}{6} \times \frac{1}{5}$ $\quad \frac{1}{2} \times \frac{1}{4} \times \frac{2}{7}$ $\quad \frac{5}{6} \times \frac{1}{2} \times \frac{2}{3}$

3. $\frac{4}{5} \times \frac{5}{6} \times \frac{3}{4}$ $\quad \frac{2}{3} \times \frac{3}{5} \times \frac{1}{8}$ $\quad \frac{3}{5} \times \frac{1}{3} \times \frac{5}{8}$

4. $\frac{3}{7} \times \frac{14}{15} \times \frac{2}{3}$ $\quad \frac{5}{6} \times \frac{9}{10} \times \frac{2}{3}$ $\quad \frac{1}{6} \times \frac{1}{2} \times \frac{3}{4}$

5. $\frac{1}{4} \times \frac{4}{5} \times \frac{5}{6}$ $\quad \frac{3}{8} \times \frac{5}{6} \times \frac{1}{10}$ $\quad \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$

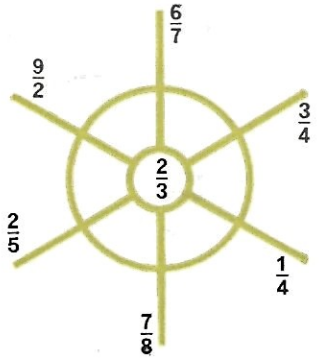
6. $\frac{3}{8} \times \frac{4}{5} \times \frac{1}{9}$ $\quad \frac{7}{15} \times \frac{3}{5} \times \frac{15}{16}$ $\quad \frac{3}{16} \times \frac{8}{9} \times \frac{3}{4}$

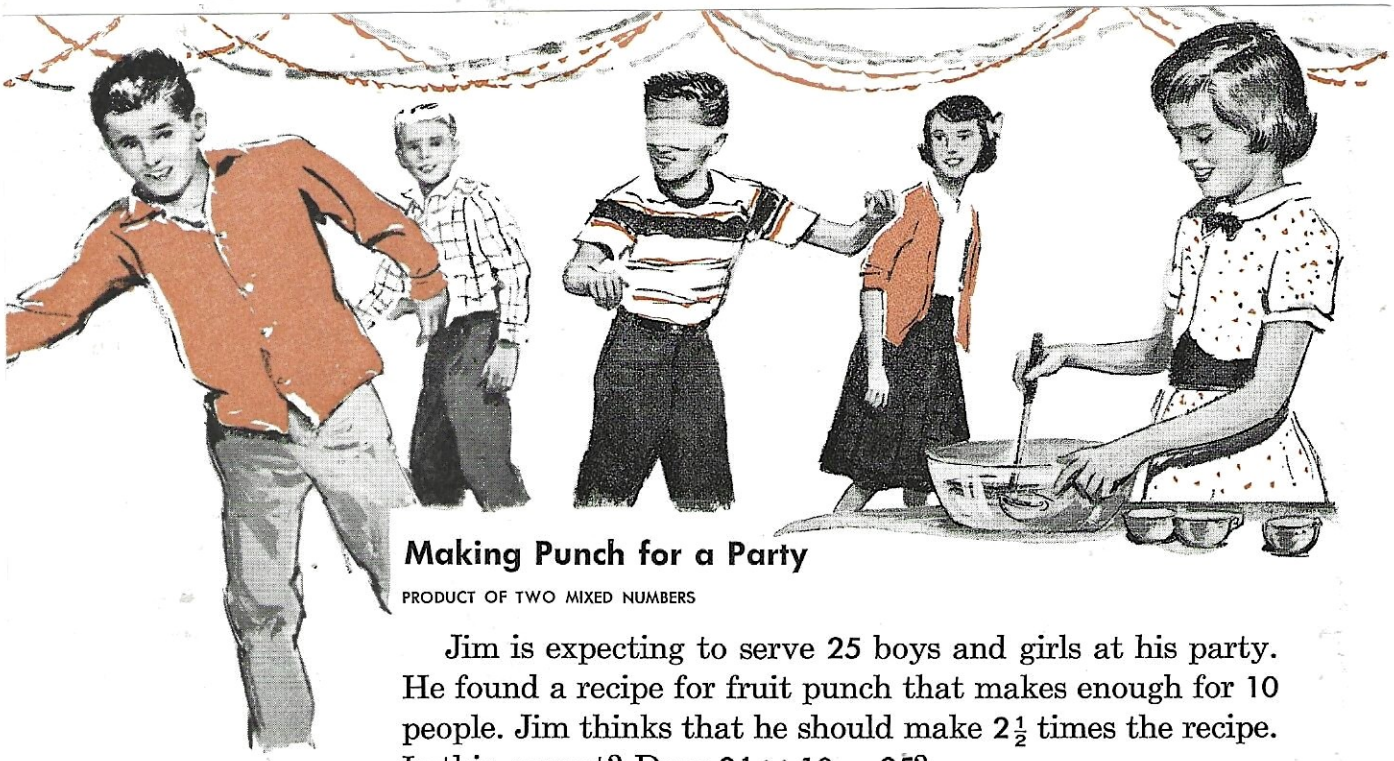
7. $\frac{7}{16} \times \frac{4}{5} \times \frac{5}{10}$ $\quad \frac{4}{5} \times \frac{5}{6} \times \frac{12}{20}$ $\quad \frac{3}{5} \times \frac{5}{6} \times \frac{10}{11}$

8. Multiply the three numbers that appear along each spoke of the wheel at the right.

   For example,

   $\frac{9}{2} \times \frac{2}{3} \times \frac{1}{4}$.





Making Punch for a Party

PRODUCT OF TWO MIXED NUMBERS

Jim is expecting to serve 25 boys and girls at his party. He found a recipe for fruit punch that makes enough for 10 people. Jim thinks that he should make $2\frac{1}{2}$ times the recipe. Is this correct? Does $2\frac{1}{2} \times 10 = 25$?

The things you use in a recipe are called *ingredients.*

1. How many different ingredients are used in this recipe?

Fruit Punch
(Serves 10)

$2\frac{1}{4}$ c. grape juice
$1\frac{3}{4}$ c. orange juice
$1\frac{1}{2}$ c. pineapple juice
$3\frac{3}{4}$ c. water
$\frac{1}{4}$ c. lemon juice
$\frac{3}{4}$ c. sugar

To find the amounts he will need for $2\frac{1}{2}$ times the recipe, Jim multiplied the amount of each ingredient by $2\frac{1}{2}$. He began with the grape juice. He thought, "How much is $2\frac{1}{2}$ times $2\frac{1}{4}$? The answer has to be more than 4, because 2×2 is 4, and $2\frac{1}{2} \times 2\frac{1}{4}$ has to be more than that. Let's see. I could multiply $2\frac{1}{2} \times 2$ and $2\frac{1}{2} \times \frac{1}{4}$, then add those two products."

2. Find the product of $2\frac{1}{2} \times 2\frac{1}{4}$ in that way.

Then Jim thought, "I could change the mixed numbers to improper fractions. Then I would have two fractions to multiply in each case."

3. Find the product of $2\frac{1}{2} \times 2\frac{1}{4}$ by completing the following:

$2\frac{1}{2} = \frac{?}{2}$ $\quad 2\frac{1}{4} = \frac{?}{4}$ $\quad \frac{5}{2} \times \frac{9}{4} = \frac{?}{8} = \underline{?}$

Jim will need $5\frac{5}{8}$ cups of grape juice.

This second method is usually the easier way to multiply two mixed numbers.

To find the product of two mixed numbers, change each to an improper fraction. Then multiply the two fractions.

4. Find the amount of each ingredient that Jim will need to make $2\frac{1}{2}$ times the recipe for fruit punch.

Practice in Multiplying Mixed Numbers and Fractions

Copy each example. Decide what a sensible answer should be: more than 1, less than the larger number being multiplied, and so forth. Then multiply. Express answers in simplest form.

1.	$1\frac{1}{2} \times 3\frac{1}{4}$	$1\frac{3}{4} \times 2\frac{1}{2}$	$1\frac{1}{2} \times 2\frac{1}{3}$	$1\frac{3}{5} \times 1\frac{1}{2}$
2.	$1\frac{1}{2} \times 1\frac{1}{5}$	$2\frac{1}{3} \times 1\frac{1}{5}$	$2\frac{1}{7} \times 1\frac{3}{4}$	$8\frac{1}{3} \times 3\frac{2}{5}$
3.	$2\frac{1}{2} \times 1\frac{1}{10}$	$1\frac{1}{2} \times 1\frac{3}{7}$	$2\frac{1}{2} \times 1\frac{9}{10}$	$2\frac{1}{2} \times 1\frac{1}{15}$
4.	$1\frac{1}{2} \times 1\frac{1}{11}$	$2\frac{3}{5} \times 3\frac{1}{3}$	$2\frac{1}{5} \times 1\frac{4}{11}$	$4\frac{3}{8} \times 5\frac{1}{5}$
5.	$2\frac{1}{2} \times \frac{3}{5}$	$4\frac{1}{2} \times \frac{2}{3}$	$5\frac{1}{4} \times \frac{1}{7}$	$5\frac{1}{4} \times \frac{4}{7}$
6.	$\frac{1}{3} \times 1\frac{3}{5}$	$\frac{1}{9} \times 3\frac{3}{5}$	$\frac{2}{7} \times 4\frac{2}{3}$	$\frac{5}{8} \times 1\frac{1}{10}$
7.	$\frac{1}{4} \times 16$	$\frac{3}{4} \times 16$	$\frac{2}{5} \times 20$	$\frac{3}{8} \times 24$
8.	$\frac{1}{2} \times 9$	$\frac{1}{6} \times 7$	$\frac{3}{8} \times 11$	$\frac{2}{3} \times 10$
9.	$14 \times \frac{1}{7}$	$12 \times \frac{5}{6}$	$5 \times \frac{3}{10}$	$4 \times \frac{3}{28}$
10.	$15 \times \frac{1}{4}$	$10 \times \frac{2}{3}$	$4 \times \frac{5}{8}$	$12 \times \frac{1}{9}$
11.	$\frac{1}{2} \times \frac{1}{4}$	$\frac{1}{2} \times \frac{1}{8}$	$\frac{1}{2} \times \frac{4}{5}$	$\frac{2}{9} \times \frac{9}{10}$
12.	$\frac{2}{3} \times \frac{6}{7}$	$\frac{3}{5} \times \frac{10}{11}$	$\frac{1}{9} \times \frac{3}{4}$	$\frac{1}{6} \times \frac{2}{5}$
13.	$\frac{4}{7} \times \frac{7}{8}$	$\frac{9}{16} \times \frac{4}{27}$	$\frac{4}{5} \times \frac{25}{28}$	$\frac{3}{10} \times \frac{20}{21}$
14.	$\frac{1}{2} \times \frac{1}{3} \times \frac{1}{5}$	$\frac{1}{2} \times \frac{2}{3} \times \frac{1}{5}$		$\frac{1}{4} \times \frac{2}{7} \times \frac{14}{15}$
15.	$\frac{5}{9} \times \frac{1}{4} \times \frac{3}{5}$	$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{5}$		$\frac{3}{7} \times \frac{14}{15} \times \frac{5}{2}$

Problems Using Fractions and Mixed Numbers

1. Alice wants to make $\frac{1}{2}$ the amount of this recipe for Deep Dish Plum Pie. Find the amount of each ingredient she will use. Then copy the recipe using $\frac{1}{2}$ of the original amounts.

Deep Dish Plum Pie

For the bottom

3 cups plums
$\frac{2}{3}$ cup sugar
$\frac{1}{2}$ teaspoon mixed spices
$\frac{1}{2}$ cup water or fruit juice

For the top

6 tablespoons butter
2 teaspoons baking powder
$\frac{1}{2}$ teaspoon salt
2 tablespoons sugar
$1\frac{1}{2}$ cups sifted flour
$\frac{1}{2}$ cup milk

2. In a newspaper, John saw an advertisement which read, "We are selling $3.85 cowhide wallets for half price." How much will John pay for a wallet if he buys one at the sale?

3. If oranges cost 39¢ a dozen, how much will Mrs. Weber pay for 8 oranges?

4. If swordfish costs 58¢ a pound, how much will $2\frac{1}{2}$ pounds of swordfish cost?

5. Frying chickens were being sold at 49¢ a pound. Mrs. Johnson bought a frying chicken that weighed $3\frac{3}{4}$ pounds. How much did she pay for it?

6. If it takes $\frac{2}{3}$ yard of cord to make one spool doll, how many yards of cord will be needed to make 5 dolls?

7. If it takes $1\frac{1}{2}$ yards of plastic lacing for one leather purse, how much lacing will you need for 3 leather purses?

8. Joan practices her piano lesson $\frac{3}{4}$ hour each day. How much time does she spend on her practicing Monday through Saturday?

9. Father drives $4\frac{3}{10}$ miles one way to work. How far does he drive in a day? In a five-day week?

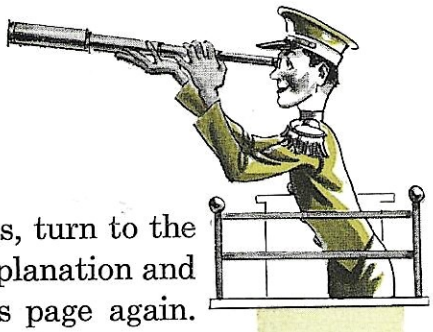
10. Jack's father owns a ranch shaped like a rectangle $\frac{3}{4}$ mile long and $\frac{1}{2}$ mile wide. What part of a square mile is the area of this ranch? See if you can draw a diagram to prove your answer.

11. How many inches are there in $\frac{1}{6}$ of $\frac{1}{3}$ of 1 yard?

12. One fluid ounce is $\frac{1}{8}$ of a cup. What part of a cup is $\frac{1}{2}$ fluid ounce?

Checking Up on Multiplying Fractions

DIAGNOSTIC TEST ON PAGES 222–242



If you make mistakes in a row of examples, turn to the page shown at the end of the row. Study the explanation and repeat the practice. Then try the row on this page again.

1.	Express the following in simplest form: $\frac{8}{24}$	$\frac{15}{5}$	$\frac{18}{4}$	$\frac{9}{9}$	222
2.	Change to improper fractions: $5\frac{1}{8}$	$7\frac{3}{5}$	$8\frac{2}{3}$	$4\frac{5}{6}$	227
	Copy. Multiply. Express answers in simplest form.				
3.	$8 \times \frac{3}{4}$	$21 \times \frac{2}{7}$	$18 \times \frac{3}{8}$	$36 \times \frac{4}{9}$	224
4.	$\frac{4}{5} \times 10$	$\frac{3}{4} \times 10$	$\frac{7}{8} \times 6$	$\frac{5}{6} \times 36$	226
5.	$2\frac{1}{2} \times 6$	$5\frac{1}{4} \times 3$	$2\frac{1}{3} \times 8$	$4\frac{1}{2} \times 12$	228
6.	$8 \times 3\frac{1}{2}$	$4 \times 4\frac{1}{4}$	$9 \times 3\frac{1}{8}$	$4 \times 8\frac{5}{6}$	230
7.	$\frac{1}{2} \times \frac{4}{5}$	$\frac{2}{3} \times \frac{3}{7}$	$\frac{3}{4} \times \frac{5}{6}$	$\frac{5}{7} \times \frac{4}{15}$	232
8.	$\frac{5}{12} \times \frac{4}{15}$	$\frac{3}{10} \times \frac{5}{9}$	$\frac{4}{21} \times \frac{7}{12}$	$\frac{3}{4} \times \frac{20}{27}$	236
9.	$\frac{1}{3} \times 3\frac{3}{5}$	$\frac{2}{5} \times 4\frac{1}{4}$	$5\frac{1}{4} \times \frac{2}{7}$	$3\frac{1}{3} \times \frac{4}{5}$	238
10.	$\frac{1}{2} \times \frac{2}{3} \times \frac{6}{7}$	$\frac{2}{3} \times \frac{3}{4} \times \frac{1}{5}$		$\frac{1}{2} \times \frac{5}{6} \times \frac{3}{10}$	241
11.	$4\frac{1}{2} \times 3\frac{1}{3}$	$2\frac{2}{5} \times 4\frac{1}{6}$	$3\frac{3}{4} \times 3\frac{1}{5}$	$4\frac{2}{3} \times 1\frac{7}{8}$	242

Ask a Question and Find the Answer

1. The average annual rainfall in the United States is 32 inches. A tropical region has $3\frac{1}{2}$ times this amount.
2. A full-grown animal weighs 96 pounds. At one year of age, it is $\frac{2}{3}$ this size.
3. The average adult sleeps $\frac{1}{3}$ of the 24 hours each day.
4. Five billion bushels of corn are raised each year throughout the world and $\frac{3}{4}$ of this is raised in the United States.

What Have You Learned about Multiplication of Fractions?

CHAPTER TEST ON MEANINGS AND SKILLS

1. An improper fraction has a _?_ that is as large as or larger than the denominator.
2. An improper fraction may be changed to a _?_ number or a _?_ number.
3. A mixed number may be changed to an _?_ fraction.
4. Add $2\frac{1}{2} + 2\frac{1}{2} + 2\frac{1}{2}$. Multiply $3 \times 2\frac{1}{2}$. Is multiplication a short way of adding?
5. Multiply $2\frac{1}{2}$ by $\frac{5}{6}$. Multiply $\frac{5}{6}$ by $2\frac{1}{2}$. Does changing the order of the multiplicand and the multiplier change the product?
6. Change the following mixed numbers to improper fractions.

   $3\frac{2}{3}$ $7\frac{4}{5}$ $9\frac{1}{2}$ $7\frac{5}{6}$ $9\frac{3}{4}$ $15\frac{1}{3}$ $17\frac{1}{2}$ $20\frac{3}{4}$
7. Change the following improper fractions to whole or mixed numbers.

   $\frac{12}{3}$ $\frac{14}{5}$ $\frac{22}{4}$ $\frac{39}{6}$ $\frac{34}{8}$ $\frac{39}{12}$ $\frac{45}{10}$ $\frac{54}{16}$
8. How can you tell that the product of $\frac{2}{5} \times \frac{5}{6}$ must be less than 1? That the product of $3\frac{3}{4} \times 1\frac{1}{6}$ must be more than 3?

Copy and multiply.

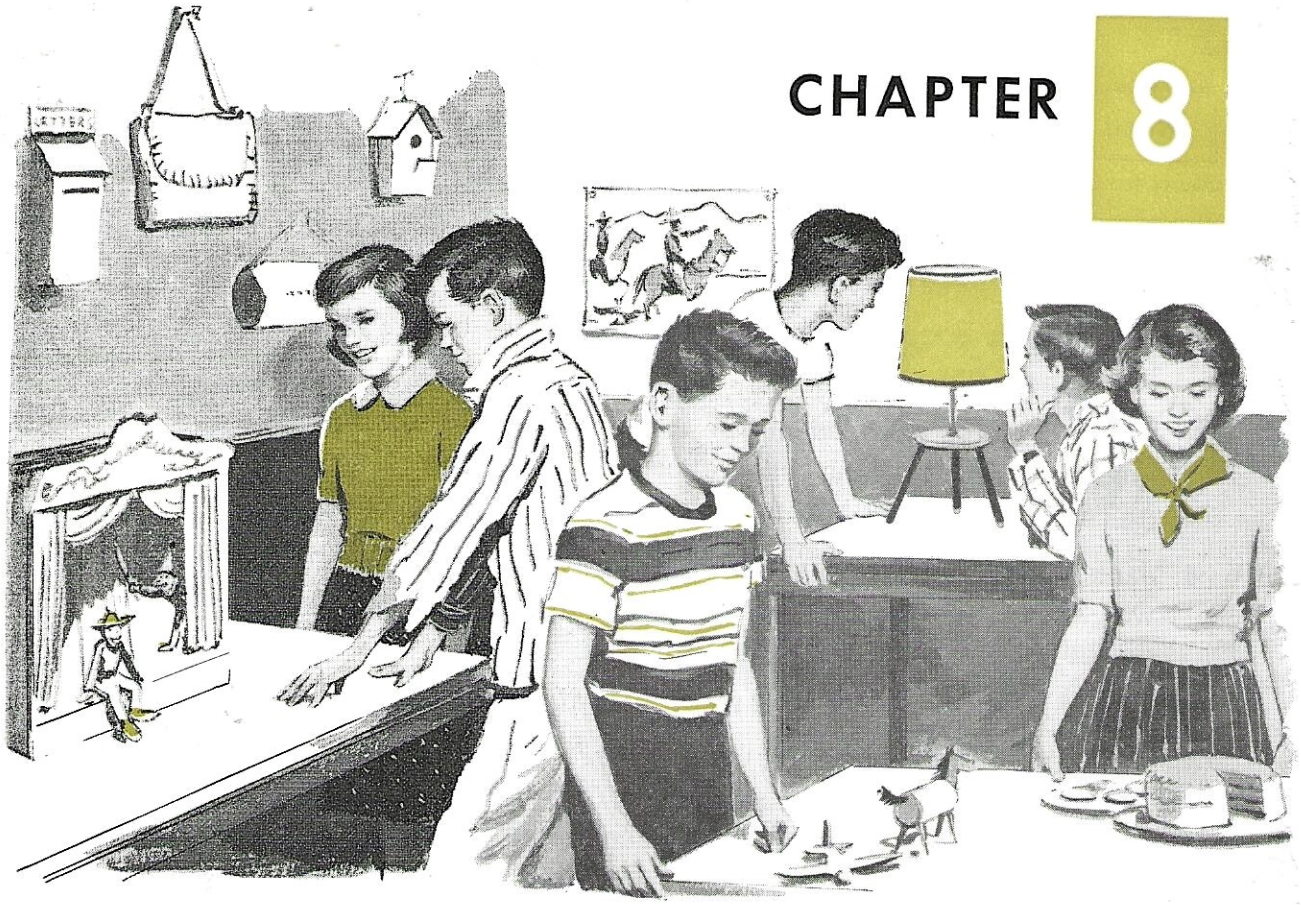
9.	$6 \times \frac{2}{3}$	$24 \times \frac{1}{7}$	$12 \times \frac{3}{8}$	$4 \times \frac{1}{6}$
10.	$\frac{1}{6} \times 24$	$\frac{3}{4} \times 14$	$\frac{1}{8} \times 2$	$\frac{2}{3} \times 9$
11.	$6 \times 2\frac{1}{2}$	$5 \times 3\frac{1}{5}$	$15 \times 4\frac{1}{8}$	$8 \times 4\frac{5}{6}$
12.	$3\frac{1}{2} \times 4$	$7\frac{1}{4} \times 3$	$3\frac{2}{3} \times 8$	$5\frac{1}{8} \times 12$
13.	$\frac{1}{2} \times \frac{1}{6}$	$\frac{2}{5} \times \frac{5}{8}$	$\frac{3}{4} \times \frac{5}{9} \times \frac{18}{25}$	$\frac{2}{5} \times \frac{5}{12} \times \frac{3}{4}$
14.	$\frac{1}{3} \times 4\frac{1}{5}$	$\frac{2}{5} \times 2\frac{1}{4}$	$3\frac{3}{7} \times \frac{5}{6}$	$4\frac{1}{2} \times \frac{2}{3}$
15.	$3\frac{1}{3} \times 2\frac{1}{5}$	$2\frac{2}{5} \times 1\frac{1}{8}$	$3\frac{3}{4} \times 1\frac{1}{6}$	$1\frac{5}{8} \times 3\frac{3}{7}$

What Have You Learned about Multiplication of Fractions?

CHAPTER TEST ON PROBLEM SOLVING

1. Lillian made 12 aprons to give as presents. Each apron required $\frac{3}{4}$ yard of cloth. How much cloth did she use for all the aprons?
2. Mrs. Smith bought 2 dozen oranges and has used $\frac{2}{3}$ of them. How many has she used?
3. The old trains traveled at the rate of 15 miles an hour. The modern train, on a long run, travels $4\frac{1}{2}$ times as fast. How many miles an hour does the modern train travel on a long run?
4. Jack picked 14 boxes of tomatoes from his garden. The average weight of a box was $8\frac{1}{2}$ pounds. How many pounds of tomatoes did he pick?
5. At $\frac{2}{5}$ of a cent per hour, how much does it cost to use an electric light bulb $\frac{5}{6}$ of an hour?
6. Mrs. Roth cooked a $3\frac{3}{4}$-pound roast of beef. At the first meal she served $\frac{1}{3}$ of the roast. How many pounds did she serve?
7. At $2\frac{2}{3}$ cents an hour, how much does it cost to use an electric roaster $4\frac{1}{2}$ hours?
8. Mrs. Morris served $\frac{1}{3}$ of a melon that weighed $5\frac{1}{4}$ pounds. What was the weight of the piece of melon she served?
9. Mrs. Brown bought $2\frac{1}{2}$ pounds of peas at 24 cents a pound. How much did she pay for the peas?
10. Mr. Morris used $2\frac{1}{2}$ bags of fertilizer for his garden. At \$2.48 a bag, what was the cost of the fertilizer he used?
11. Mrs. Noble used $2\frac{1}{4}$ pounds of sugar one week and $1\frac{7}{8}$ pounds the next week. At 8 cents a pound, what was the cost of the sugar she used?
12. Mrs. Roth allows 20 minutes for each pound in roasting beef. How many minutes did she require to roast $3\frac{3}{4}$ pounds of beef? Change your answer into hours and minutes.

CHAPTER 8



The Hobby Show

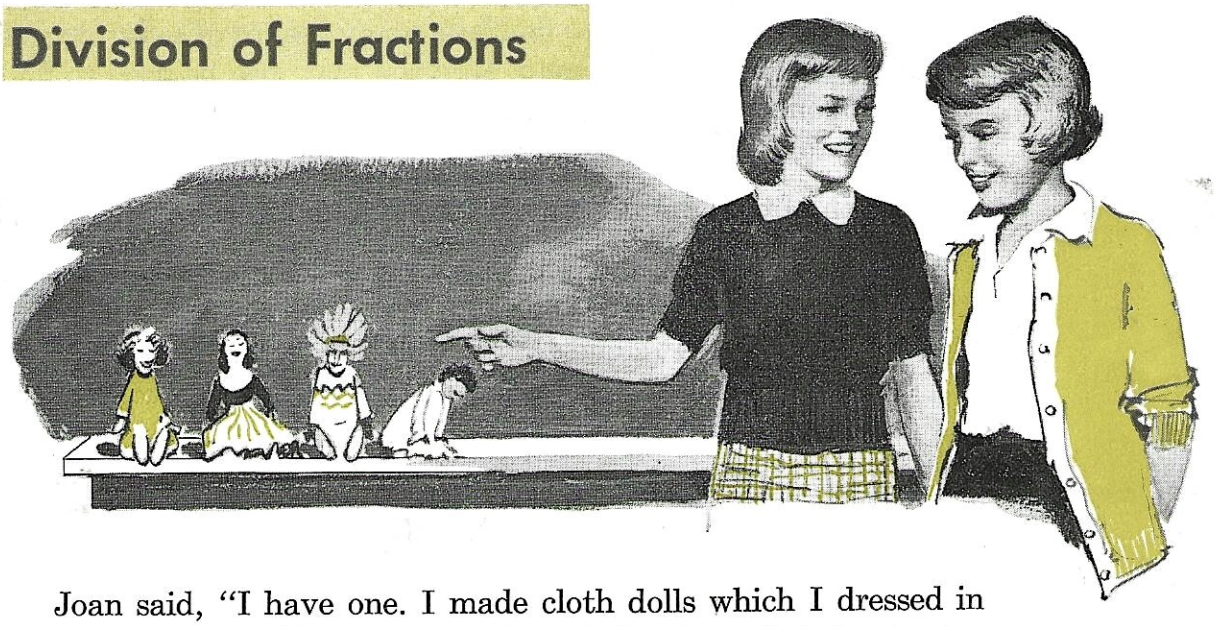
THINKING ABOUT DIVISION

Miss White's sixth graders formed a Hobby Club. In the spring, they had a show so that everyone could see what the club members had made. The picture shows a part of their exhibit.

The pupils found that they used a great deal of arithmetic in their work with crafts, sewing, and cooking. They were discussing this in arithmetic class. Some of them had figured out ways to divide fractions in many different kinds of problems.

Miss White said, "Let's think about what we know about dividing and about fractions. Then each of you can explain to the class how you solved a problem in which a fraction appeared in the dividend, in the divisor, or in both. Who used a problem in dividing whole numbers?"

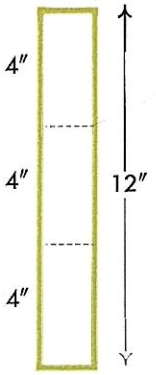
Division of Fractions



Joan said, "I have one. I made cloth dolls which I dressed in various costumes. For each arm, I needed a strip of cloth 4 inches long. How many arms could I make out of a strip of cloth 12 inches long?"

Think of all the ways Joan might find her answer.

1. Could she draw a picture? How would she find the answer from the picture?
2. Could she find the answer by adding? How would she do that?
3. Could she find the answer by subtracting? Explain.
4. Could she find the answer by thinking, "$\_?\_ \times 4$ in. $= 12$ in."?
5. Could she find the answer by dividing? Is this the easiest way to find the answer, if you know how to divide?



Below are three ways to write the division example.

$$4\text{ in.}\overline{)12\text{ in.}}\qquad 3 \qquad 12\text{ in.} \div 4\text{ in.} = 3 \qquad \frac{12\text{ in.}}{4\text{ in.}} = 3$$

6. In each of the three ways, what is the divisor? What is the dividend? What is the quotient? What does the quotient tell? Is the answer 3 inches, or just 3?

Division may be used to find how many groups of a certain size are contained in a larger group. Division may also be used to find the size of one part when something is shared equally or in any way divided into a number of equal small groups.

7. Which of the two uses for division is found in Joan's problem?

8. Make up some other problems that call for the same use of division. Choose one of your problems in which the numbers are not very large. Pretend that you do not know how to divide. Find the correct answer to that problem in as many other ways as you can think of.

9. Could you use the same ways to find the answer if the divisor were a fraction? Try it with the following examples.

$3 \div \frac{1}{2}$ $\quad$ $8 \div \frac{1}{4}$ $\quad$ $6 \div \frac{2}{3}$ $\quad$ $3\frac{1}{2} \div \frac{7}{8}$

Ken said, "I had a problem that called for the other use of division. I was making a wrought-iron lamp base that has three legs, and my mother wanted me to make it as tall as possible with the metal that I had. I had a metal rod 28 inches long."

10. Can you think of some ways to divide 28 of anything into 3 equal parts without using the division facts?

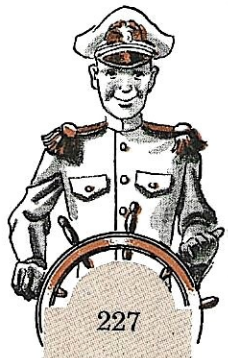
Ken said, "I knew that $28 \div 3 = 9\frac{1}{3}$, but that did not do me any good, because I did not have a ruler that measured thirds of an inch. I had to think of another way to find out where to cut my metal rod. Finally I got an idea that worked. I cut a strip of paper the same length as the rod, and folded that paper into three equal parts. Then I used the folds in the paper to mark the places to cut the rod."

11. Could you use Ken's idea of folding the paper to find the answer to a problem that had a fraction in the dividend? Try it with these examples. Use your ruler to make the strips the correct lengths and to tell what the answer is.

$\frac{3}{4}'' \div 2$ $\quad$ $2\frac{1}{2}'' \div 4$ $\quad$ $4\frac{1}{2}'' \div 3$ $\quad$ $10\frac{1}{2}'' \div 2$

Are You Ready for Division of Fractions?

READINESS TEST



If you can do the work on this page, you are ready for division of fractions.

1. Change the following to improper fractions. $1\frac{4}{5}$ $4\frac{2}{9}$ $6\frac{1}{10}$ — 227

2. Find the answers to each of the following examples. What can you say about the answers in row (a)? in row (b)? — 223

(a) $\frac{1}{3}$ of 12	$\frac{1}{3} \times 12$	$12 \times \frac{1}{3}$	$12 \div 3$	
(b) $\frac{1}{5}$ of 2	$\frac{1}{5} \times 2$	$2 \times \frac{1}{5}$	$2 \div 5$	

3. In each of the following divisions, what number is the dividend? What number is the divisor? What number is the quotient? — 48

(a) $4\overline{)12}$ (b) $14 \div 7 = 2$ (c) $\frac{30}{5} = 6$

4. How do you check a division example? — 50

5. Check each of the following division examples. — 50

(a) $2\overline{)5}$ (b) $18 \div 6 = 3$ (c) $8 \div 10 = \frac{4}{5}$

Copy and multiply.

6. $3 \times \frac{5}{3}$	$4 \times \frac{2}{5}$	$\frac{2}{7} \times 14$	$\frac{3}{8} \times 6$	225, 226
7. $\frac{1}{2} \times \frac{1}{3}$	$\frac{2}{3} \times \frac{6}{7}$	$2\frac{1}{2} \times \frac{1}{5}$	$\frac{2}{3} \times 4\frac{1}{4}$	232, 238
8. $1\frac{1}{2} \times 1\frac{1}{5}$	$2\frac{1}{7} \times 3\frac{1}{2}$	$3\frac{1}{3} \times 5\frac{2}{5}$	$1\frac{1}{4} \times 1\frac{3}{5}$	242

9. In which of the following examples will the answer be greater than 1? Equal to 1? Less than 1? How can you tell? — 49, 89

(a) $12 \div 4$ (b) $12 \div 18$ (c) $12 \div 12$

If you made mistakes in any row, turn to the page listed beside the row. Study the explanation given there. Then work on the practice examples until you are sure you can do them correctly.

Dividing a Whole Number by a Unit Fraction

A fraction whose numerator is 1 is called a *unit fraction*. It is easy to divide by a unit fraction if you think of what you know about fractions and about the meaning of division.

Study the following problems.

1. Bob wants to make some potato prints. He needs $\frac{1}{2}$ potato for each print. How many prints can he make with 3 potatoes?
   (a) Draw a picture to show how many halves there are in 3.
   (b) How many halves would you have to add to get 3?
   (c) Beginning with 3, how many times can you subtract $\frac{1}{2}$?
   (d) $\_?\_ \times \frac{1}{2} = 3$.
   (e) There are $\_?\_$ halves in 1 potato. In 3 potatoes there are 3 times as many halves, or $\_?\_$ halves.
   (f) All of these ways of getting the answer show that $3 \div \frac{1}{2} = \_?\_$.

2. Jane cut 4 sandwiches into halves. How many halves are there in 4? In 1 sandwich there are $\_?\_$ halves, so in 4 sandwiches there will be 4 times as many, or $\_?\_$. $4 \div \frac{1}{2} = \_?\_$. Does $4 \div \frac{1}{2} = 4 \times 2$?

3. Each strand of hair that Joan puts on her cloth dolls takes $\frac{1}{6}$ yard of yarn. How many strands can Joan make with 4 yards of yarn?

To find her answer, Joan must find how many sixths there are in 4. There are $\_?\_$ sixths in 1, so in 4 there will be 4 times as many sixths, or $\_?\_$. $4 \div \frac{1}{6} = \_?\_$. Does $4 \div \frac{1}{6} = 4 \times 6$?

Use any method that makes sense to you to answer these questions.

4. How many halves are there in 2? 3? 5? 6? 8? 10? Compare your answers with the original numbers. Can you tell an easy way to find how many halves there are in any whole number?

5. How many thirds are there in 1? 2? 5? 7? 9? Compare your answers with the original numbers. Can you tell an easy way to find how many thirds there are in any whole number?

6. Dividing by $\frac{1}{2}$ gives the same result as multiplying by __?__. Dividing by $\frac{1}{3}$ gives the same result as multiplying by __?__. Dividing by $\frac{1}{4}$ should give the same result as multiplying by __?__. Dividing by any unit fraction gives the same result as multiplying by its __?__.

7. Make a rule for dividing a whole number by a unit fraction.

Use your rule to find the answers to the following divisions.

8.	$2 \div \frac{1}{2}$	$7 \div \frac{1}{2}$	$3 \div \frac{1}{2}$	$6 \div \frac{1}{4}$	$5 \div \frac{1}{4}$
9.	$4 \div \frac{1}{8}$	$3 \div \frac{1}{8}$	$5 \div \frac{1}{8}$	$2 \div \frac{1}{5}$	$3 \div \frac{1}{5}$
10.	$7 \div \frac{1}{3}$	$8 \div \frac{1}{3}$	$4 \div \frac{1}{6}$	$9 \div \frac{1}{6}$	$2 \div \frac{1}{7}$
11.	$2 \div \frac{1}{10}$	$5 \div \frac{1}{10}$	$3 \div \frac{1}{9}$	$8 \div \frac{1}{9}$	$20 \div \frac{1}{4}$

Look at your work for examples 8–11.

12. The dividend is the number before the division sign. The number after the division sign is the __?__.

13. When you divide by a unit fraction, is the answer larger or smaller than the dividend?

14. How do you check a division example in which the dividend and the divisor are both whole numbers?

15. Would you expect the same way of checking to work if the divisor were a fraction? Try it with the following example.

Dividend ÷ divisor = quotient and quotient × divisor = dividend.

$3 \div \frac{1}{8} = 24$ $\qquad$ $24 \times \frac{1}{8} =$ __?__

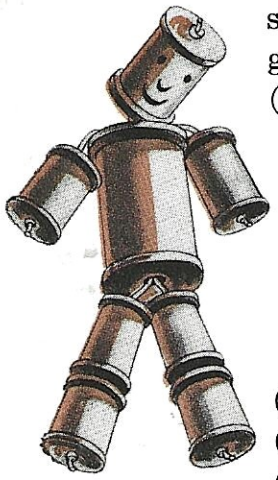
Solve the following problem. Check the result.

16. Jack is putting turtle decorations on the paper weights he is making. For each turtle he uses $\frac{1}{2}$ of an English walnut shell. How many turtles can he make with 19 English walnuts?

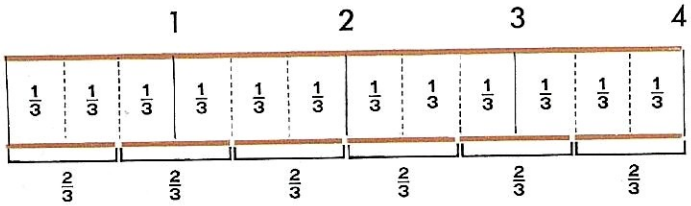
Dividing a Whole Number by a Multiple Fraction

A fraction whose numerator is more than 1 is called a ***multiple fraction.***

1. To fasten together the body of each spool doll Alice is making, she needs $\frac{2}{3}$ yard of string. How many dolls can she fasten together if she has 4 yards of string?



(a) Find the answer by using the picture below.



(b) How many $\frac{2}{3}$'s would you have to add to get 4?

(c) Beginning with 4, how many times can you subtract $\frac{2}{3}$?

(d) $\_?\_ \times \frac{2}{3} = 4$.

(e) There are 12 thirds in 4. If these 12 thirds are grouped by 2's so that there are 2 thirds in each group, there will be $12 \div 2$, or $\_?\_$ groups.

(f) All of the above ways of getting the answer show that $4 \div \frac{2}{3} = \_?\_$.

(g) Use fraction cutouts or a diagram to find out how many two-thirds there are in 1. Are there $1\frac{1}{2}$ (or $\frac{3}{2}$) two-thirds in 1? Would there be 4 times as many in 4?

(h) Does $4 \div \frac{2}{3}$ give the same result as $4 \times \frac{3}{2}$?

2. Jean is making tea towels. For each towel, she uses $\frac{3}{4}$ yard of toweling. How many towels can she make from 3 yards of toweling? Use any method that makes sense to you to find the answer. $3 \div \frac{3}{4} = \_?\_$. Does $3 \div \frac{3}{4}$ give the same result as $3 \times \frac{4}{3}$?

3. Look back at your results for problems 1 and 2. Dividing by $\frac{2}{3}$ gives the same result as multiplying by $\_?\_$. Dividing by $\frac{3}{4}$ gives the same result as multiplying by $\_?\_$. Dividing by $\frac{5}{8}$ should give the same result as multiplying by $\_?\_$.

Study the next four lines and see whether they show the reason for your answers to exercise 3.

4. Does multiplying both the divisor and the dividend by the same number change the quotient?

5. Any number $\div 1 =$ _?_. $\frac{5}{6} \times$ _?_ $= 1$.

$$30 \div \tfrac{5}{6} = (30 \times \tfrac{6}{5}) \div (\tfrac{5}{6} \times \tfrac{6}{5}) = (30 \times \tfrac{6}{5}) \div 1 = 30 \times \tfrac{6}{5} = 36$$

When the product of two numbers is 1, each number is the *reciprocal* of the other. Since $\frac{6}{5} \times \frac{5}{6} = 1$, $\frac{6}{5}$ is the reciprocal of $\frac{5}{6}$ and $\frac{5}{6}$ is the reciprocal of $\frac{6}{5}$. The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. The reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$, or 2. Another name for *reciprocal* is *multiplicative inverse.*

6. Find the reciprocal of the following fractions.

$\frac{2}{3}$ $\frac{5}{6}$ $\frac{1}{8}$ $\frac{9}{10}$ $\frac{2}{7}$ $\frac{3}{5}$ $\frac{4}{3}$ $\frac{6}{7}$ $\frac{8}{11}$ $\frac{3}{2}$ $\frac{5}{4}$ $\frac{1}{6}$

Remember that the divisor is the number after the $\div$ sign. Then *dividend* $\div$ *divisor* = *dividend* $\times$ *reciprocal of divisor* gives us a good rule for dividing by a fraction.

> To divide a number by a fraction, multiply by the reciprocal of the fraction.

Divide.

7. $30 \div \frac{5}{6}$	$15 \div \frac{3}{5}$	$3 \div \frac{2}{3}$	$2 \div \frac{3}{4}$
8. $24 \div \frac{3}{8}$	$8 \div \frac{2}{5}$	$8 \div \frac{3}{4}$	$9 \div \frac{2}{3}$
9. $12 \div \frac{2}{3}$	$9 \div \frac{3}{4}$	$5 \div \frac{4}{5}$	$12 \div \frac{5}{6}$

10. In examples 7–9, all the divisors are proper fractions. If you find the reciprocal of a proper fraction and multiply this reciprocal and the dividend, will the answer be larger than the dividend? Why should this be so? Are your answers to those examples all larger than the dividends? This is one way of telling whether your answer is sensible.

Dividing a Whole Number by Any Fraction

1. How many $\frac{1}{4}$'s are there in 1? in 2? $2 \div \frac{1}{4} = 2 \times$ _?_ $= 8$.
2. What is the reciprocal of $\frac{1}{4}$? Does $\frac{4}{1}$ have the same value as 4? Would the rule about multiplying by the reciprocal apply to unit fractions too?
3. Use some way besides dividing to find how many $\frac{3}{2}$'s there are in 6. Then work $6 \div \frac{3}{2}$ by multiplying by the reciprocal of the divisor. Did you get the same answer? Did you expect to? What kind of fraction is $\frac{3}{2}$? Will the rule about using the reciprocal apply to all kinds of fractions? Try some other examples if you are not sure.
4. If the fraction you are dividing by has a value less than 1, will you be multiplying by a number greater than 1 when you use its reciprocal? Then will your answer be larger or smaller than the dividend?
5. If the fraction you are dividing by has a value greater than 1, will you be multiplying by a number less than 1 when you use its reciprocal? Then your answer will be _?_ than the dividend.

Divide. Be sure your answers are sensible.

6.	$4 \div \frac{1}{3}$	$3 \div \frac{1}{6}$	$7 \div \frac{1}{8}$	$8 \div \frac{1}{4}$
7.	$9 \div \frac{1}{7}$	$6 \div \frac{2}{3}$	$15 \div \frac{5}{6}$	$5 \div \frac{5}{7}$
8.	$12 \div \frac{3}{4}$	$14 \div \frac{7}{8}$	$20 \div \frac{5}{8}$	$8 \div \frac{3}{4}$
9.	$9 \div \frac{3}{5}$	$8 \div \frac{2}{5}$	$40 \div \frac{5}{9}$	$4 \div \frac{4}{3}$
10.	$5 \div \frac{5}{2}$	$6 \div \frac{6}{5}$	$12 \div \frac{4}{3}$	$16 \div \frac{8}{7}$
11.	$7 \div \frac{2}{3}$	$8 \div \frac{5}{6}$	$5 \div \frac{2}{5}$	$4 \div \frac{3}{2}$

12. Find the answer to $3 \div \frac{2}{3}$. Draw a diagram to show that your answer means there are 4 whole two-thirds and $\frac{1}{2}$ of another two-thirds in 3.

13. In checking a division by multiplying the quotient and the divisor, should you use the original divisor or its reciprocal? Why?

14. Check your answers to examples in rows 7 and 9.

Dividing a Whole Number by a Mixed Number

You know how to divide a whole number by any fraction. You know how to change a mixed number to an improper fraction. Does that give you an idea of how to divide a whole number by a mixed number? See if you can solve the following problem without help.

1. Bill needs $1\frac{1}{2}$ yards of plastic lacing for each coin purse he is making. How many coin purses can he finish with 6 yards of lacing? $6 \div 1\frac{1}{2} = 6 \div \frac{?}{2} = 6 \times \frac{?}{\quad} = \frac{?}{\quad}$

2. What number is the dividend in the example you used for problem 1? Is the answer larger or smaller than the dividend? Why?

3. Study the following examples. Then copy them without the work. Close your book and do the examples. Compare your work with the work in the book.

$10 \div 2\frac{1}{2} =$	$5 \div 1\frac{1}{3} =$	$2 \div 3\frac{1}{4} =$
$10 \div \frac{5}{2} =$	$5 \div \frac{4}{3} =$	$2 \div \frac{13}{4} =$
$\overset{2}{\cancel{10}} \times \frac{2}{\cancel{5}_{1}} = 4$	$5 \times \frac{3}{4} = \frac{15}{4} = 3\frac{3}{4}$	$2 \times \frac{4}{13} = \frac{8}{13}$

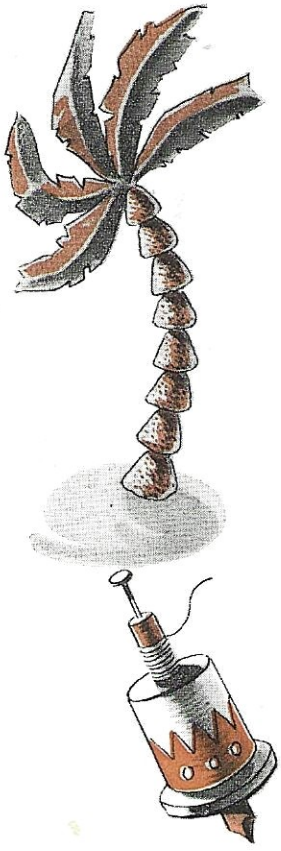
4. When a whole number is divided by a mixed number, may the answer be a whole number? May it be a mixed number? A proper fraction? Is the answer always smaller than the dividend?

5. Make up a rule for dividing a whole number by a mixed number. Then turn the page.

Use your rule to do the following divisions. In each one, decide whether your answer is sensible. Express all answers in simplest form.

6.	$5 \div 1\frac{1}{4}$	$12 \div 1\frac{1}{5}$	$8 \div 2\frac{2}{3}$	$9 \div 1\frac{1}{2}$
7.	$5 \div 1\frac{2}{3}$	$4 \div 3\frac{1}{7}$	$8 \div 2\frac{1}{2}$	$7 \div 3\frac{2}{3}$
8.	$12 \div 4\frac{4}{5}$	$10 \div 2\frac{2}{9}$	$20 \div 6\frac{2}{3}$	$36 \div 2\frac{1}{4}$
9.	$2 \div 2\frac{1}{8}$	$4 \div 5\frac{1}{3}$	$6 \div 7\frac{3}{4}$	$5 \div 6\frac{2}{5}$

Problems to Solve



1. Mrs. Brown has 3 grapefruit. If she wants to give each person $\frac{1}{2}$ grapefruit, how many people can she serve?
2. Each fruit favor that Ann makes requires $\frac{1}{4}$ yard of ribbon to put it together. How many favors can Ann put together if she has 4 yards of ribbon?
3. Joe read that he will need $\frac{1}{3}$ yard of wire to make one palm tree. How many palm trees can Joe make if he has 2 yards of wire?
4. Alice needs $\frac{1}{2}$ yard of string to fasten each pair of arms on her spool dolls. How many pairs of arms can be fastened if Alice has 4 yards of string to use?
5. How many towels $\frac{3}{4}$ yard long can Ruth make from 9 yards of toweling?
6. Bill makes necktie holders. If it takes him $\frac{2}{3}$ of an hour to cut one with his jig saw, how many can he cut in 4 hours?
7. It takes $\frac{5}{6}$ yard of material to make a laundry bag. How many laundry bags can be made from 10 yards of material?
8. How many spool tops can Jim make if it takes $\frac{2}{3}$ yard of string for each top and Jim has 6 yards of string?

9. How many $\frac{3}{4}$ inch strips of tape can Jim cut from 6 inches of tape?

10. To put together the body of a wire doll, Mary needs $1\frac{1}{3}$ feet of wire. How many dolls can she put together if she has 8 yards of wire?

11. Each comb case that Bob is making requires $1\frac{3}{4}$ yards of lacing to put it together. How many can Bill put together if he has 7 yards of lacing to work with?

12. Ellen is binding blankets for her mother. How many $4\frac{1}{2}$ inch wide strips can she cut from a piece of satin 36 inches wide?

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–7)

1. Which of the following numbers means 7 thousands, 6 hundreds, 4 tens, 5 ones? 6457 7546 7645 5467
2. Write the Roman numerals for: 54 301 647 927
3. Find the area of a rectangle 15 in. by 18 in.
4. Find the perimeter of a triangle with sides 10″, 8″, and 5″.
5. Add: 4 lb. 12 oz. and 6 lb. 14 oz. Divide this sum by 2.
6. Subtract: 5′ 11″ from 8′ 7″. Multiply the difference by 4.
7. 12 gal. = _?_ qt. 360 min. = _?_ hr. 432 sq. in. = _?_ sq. ft.
8. Find the sum of 34,104 + 8,249 + 62,310 + 4,200 + 796.
9. Subtract: 7642 from 9801; \$192.76 from \$382.14.

Copy. Multiply. Check your work.

10.

374	762	900	3409	648
58	346	768	86	305

Turn to the next page.

Copy. Divide. Check your work.

11. $43\overline{)1548}$ $92\overline{)4876}$ $63\overline{)38241}$ $38\overline{)1996}$ $69\overline{)25464}$

Copy. Do as the signs tell you.

12. $17\frac{3}{4} + 6\frac{3}{8}$ $14\frac{5}{6} + 12\frac{3}{4}$ $6\frac{1}{3} + 7\frac{1}{2}$ $12\frac{3}{5} + 4\frac{1}{4}$

13. $8\frac{3}{4} - 2\frac{1}{2}$ $9\frac{1}{3} - 3\frac{5}{6}$ $7 - 1\frac{7}{8}$ $14\frac{1}{3} - 8\frac{1}{2}$

14. $5 \times \frac{1}{3}$ $\frac{1}{4} \times 28$ $6 \times 3\frac{1}{2}$ $\frac{1}{6} \times \frac{3}{7}$

15. $\frac{8}{9} \times \frac{3}{4}$ $2\frac{1}{4} \times 3$ $\frac{3}{5} \times 3\frac{1}{3}$ $2\frac{2}{5} \times 3\frac{1}{6}$

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–7)

Read each problem carefully. Decide when to add, subtract, multiply, or divide. Then find the answer.

1. When Mr. Marshall entered a tunnel the mileage meter registered 3297.6 miles. When he left the tunnel, it registered 3303.8 miles. How long was the tunnel?
2. On a trip to the South, the Marshall family traveled 285.6 miles the first day, 256.4 miles the second day, and 198.8 miles the third day. How many miles in all did they travel in the three days?
3. Mr. Marshall's house is built on a rectangular lot 153 feet long and 85 feet wide. What is the area of the lot? What is the perimeter?
4. It took Mr. Kane from 6:00 P.M. to 11:00 P.M. to read a book of 360 pages. What was the average number of pages he read in an hour?
5. Last week Nancy practiced $3\frac{1}{2}$ hours on her piano lessons. This week she practiced $2\frac{3}{4}$ hours. How many more hours did she practice last week than this?

6. On a fishing trip Jim caught a trout that weighed 2 pounds 9 ounces. His father caught one that weighed 3 pounds 2 ounces. What was the difference in weight of these fish?

7. Julia used 6 feet 7 inches of red ribbon and 5 feet 9 inches of blue ribbon to make Easter decorations. How much ribbon did she use in all?

8. Mr. Knox bought a new car for \$2520. He made a down payment of \$1740. He will take care of the remainder in 12 equal payments. How much is each payment?

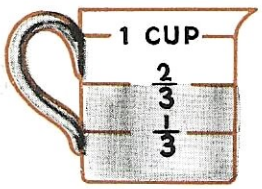
9. Ted picked 16 boxes of beans holding an average weight of $2\frac{3}{8}$ pounds. How many pounds of beans did he pick?

10. If it costs Mrs. Smith $3\frac{1}{3}$ cents an hour to use her electric iron, how much will it cost her to use it for $4\frac{1}{2}$ hours?

11. Mrs. Smith bought $3\frac{3}{4}$ pounds of meat at 52 cents a pound. How much did she pay for the meat? How much change did she receive from the 5 dollars she gave the clerk?

12. The senior campers went on a hike. The distance to the first stopping place was $2\frac{3}{4}$ miles, to the second stopping place $3\frac{1}{2}$ miles, and to the last stopping place $4\frac{1}{4}$ miles. How many miles long was the hike?

Thinking about Dividing a Fraction



1. Judith is making cookies. The recipe tells her to add the milk in 2 equal parts. How much milk will she add each time if the recipe calls for $\frac{2}{3}$ cup of milk?
   (a) Use the drawing at the left to find the answer. $\frac{2}{3} \div 2 =$?.
   (b) Is dividing a number into 2 equal parts the same as finding $\frac{1}{2}$ of the number? $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{2} \times \frac{2}{3} =$?.
   (c) Do $\frac{2}{3} \times \frac{1}{2}$ and $\frac{1}{2} \times \frac{2}{3}$ have the same product?
   (d) Is there a way to write a whole number as a fraction? Can you think of a way to change $\frac{2}{3} \div 2$ to $\frac{2}{3} \times \frac{1}{2}$?

2. Find the reciprocal of the following fractions.

   $\frac{2}{3}$ $\frac{1}{3}$ $\frac{3}{1}$ $\frac{1}{4}$ $\frac{4}{1}$ $\frac{5}{6}$ $\frac{8}{1}$ $\frac{7}{8}$ $\frac{9}{1}$ $\frac{9}{10}$

3. Write the following whole numbers as fractions with a denominator of 1. Then find the reciprocals.

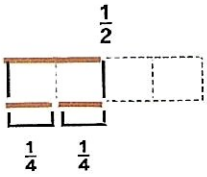
   5 6 3 2 4 8 16 12 9 21

4. Use any way you know to find the answer to $\frac{1}{4} \div 2$. Is the answer smaller than $\frac{1}{4}$?

5. Find the answer to $\frac{3}{5} \div 3$. Is the answer smaller than $\frac{3}{5}$?

6. What numbers are the dividends in the division examples in questions 4 and 5? When any fraction is divided by a whole number, will the answer be smaller than the dividend?

Dividing a Fraction by a Whole Number



1. Jane has $\frac{1}{2}$ yard of wire to use for arms on a wire doll. If she cuts the $\frac{1}{2}$ yard into 2 equal parts, how long will each piece be?

Jane drew a diagram like the one at the left. She found that $\frac{1}{2}$ divided into 2 equal parts is $\frac{1}{4}$.

Amy said, "What you are trying to find is how much $\frac{1}{2}$ of $\frac{1}{2}$ is. I would find the answer this way:

$$\frac{1}{2} \div 2 = \frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$"

Joan said, "Why can't we use the rule we learned when we divided a whole number by a fraction? We could think of 2 as $\frac{2}{1}$, if we wanted to make it look like a fraction. I would find the answer this way:

$$\frac{1}{2} \div 2 = \frac{1}{2} \div \frac{2}{1} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$"

Did all three girls get the same answer? When you divide a fraction by a whole number, will the answer always be smaller than the dividend?

2. John wants to cut $\frac{3}{4}$ of a large sheet of oak tag into 6 equal parts. What part of the whole sheet will each piece be?
   (a) Find the answer by finding $\frac{1}{6}$ of $\frac{3}{4}$.
   (b) Find $\frac{3}{4} \div 6$ by thinking of 6 as $\frac{6}{1}$. Then use the rule for dividing by a fraction.
   (c) Look at the two examples you just did. Does $\frac{1}{6} \times \frac{3}{4}$ make the same amount as $\frac{3}{4} \times \frac{1}{6}$?
   (d) Can you use the rule about multiplying by the reciprocal of the divisor when the divisor is a whole number?
   (e) Is the reciprocal of a whole number always a unit fraction?
   (f) Make a rule for finding the reciprocal of a whole number. Use your rule to give the reciprocal of 2, of 3, of 5, of 123, of 469, of n.

Divide. Give quotients in lowest terms.

3.	$\frac{1}{3} \div 5$	$\frac{1}{8} \div 3$	$\frac{2}{3} \div 4$	$\frac{1}{5} \div 2$	$\frac{5}{6} \div 2$
4.	$\frac{3}{5} \div 2$	$\frac{1}{2} \div 3$	$\frac{3}{8} \div 2$	$\frac{5}{7} \div 3$	$\frac{3}{10} \div 2$
5.	$\frac{2}{3} \div 3$	$\frac{3}{4} \div 2$	$\frac{1}{2} \div 6$	$\frac{2}{3} \div 5$	$\frac{3}{4} \div 3$
6.	$\frac{4}{5} \div 2$	$\frac{1}{3} \div 3$	$\frac{5}{6} \div 3$	$\frac{1}{4} \div 4$	$\frac{2}{5} \div 4$
7.	$\frac{1}{4} \div 2$	$\frac{1}{5} \div 5$	$\frac{3}{5} \div 4$	$\frac{3}{4} \div 5$	$\frac{3}{8} \div 4$

Dividing a Mixed Number by a Whole Number

You know how to divide a fraction by a whole number. You know how to change a mixed number to an improper fraction. Does that give you an idea of how to divide a mixed number by a whole number? Think about it. Then see if you can solve the following problem without help.

1. Joe has $4\frac{1}{2}$ yards of plastic lacing. How much lacing can he use for each coin purse if he has 3 coin purses to lace? $4\frac{1}{2} \div 3 = \frac{?}{2} \div 3 = \underline{?}$.

2. If a division is correct, the product of the quotient and the divisor will equal the dividend. In the division example you used for problem 1, what number is the dividend? The divisor? The quotient? Does $3 \times 1\frac{1}{2} = 4\frac{1}{2}$?

3. Study the following examples. Copy them without the work. Close your book and do the examples and the checks. Compare your work with the work in the book.

$7\frac{1}{2} \div 9 = \frac{15}{2} \div \frac{9}{1} = \frac{\overset{5}{\cancel{15}}}{2} \times \frac{1}{\underset{3}{\cancel{9}}} = \frac{5}{6}$ $\qquad$ $\frac{\overset{5}{\cancel{10}}}{\underset{2}{\cancel{6}}} \times \frac{\overset{3}{\cancel{9}}}{1} = \frac{15}{2} = 7\frac{1}{2}$

$3\frac{1}{3} \div 2 = \frac{10}{3} \div \frac{2}{1} = \frac{\overset{5}{\cancel{10}}}{3} \times \frac{1}{\underset{1}{\cancel{2}}} = \frac{5}{3} = 1\frac{2}{3}$ $\qquad$ $1\frac{2}{3} \times 2 = \frac{5}{3} \times 2 = \frac{10}{3} = \underline{?}$

Do the following examples and problems. Check your work.

4. $5\frac{1}{2} \div 4$ $\qquad$ $2\frac{2}{5} \div 4$ $\qquad$ $4\frac{2}{3} \div 2$ $\qquad$ $3\frac{3}{6} \div 6$

5. $2\frac{1}{4} \div 6$ $\qquad$ $6\frac{1}{4} \div 5$ $\qquad$ $4\frac{3}{8} \div 5$ $\qquad$ $3\frac{3}{5} \div 3$

6. $4\frac{1}{5} \div 7$ $\qquad$ $5\frac{1}{6} \div 3$ $\qquad$ $2\frac{1}{12} \div 5$ $\qquad$ $2\frac{2}{15} \div 4$

7. John has to fill 4 boxes from $2\frac{1}{2}$ pounds of stuffed prunes. What part of a pound should he put in each box?

8. John is helping to pack some fancy apples in boxes. From a peck of apples weighing $12\frac{1}{2}$ pounds he wishes to pack 5 boxes. If each box has an equal weight of apples, how many pounds should go into each box?

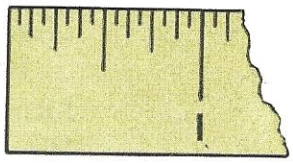
Just for Fun

See if you can answer the following questions.

1. The John Hancock Building in Boston weighs about 140,000 tons. Is that more than a million pounds? As much as a billion pounds?
2. There are 8,300,000 feet of electric wiring in the building. How many miles of wiring is that?
3. The inter-office telephone lines complete about $3\frac{1}{2}$ million calls a year. How do you write $3\frac{1}{2}$ million without using a fraction?
4. The elevators run at the rate of 800 feet per minute. How many feet per hour is that? How many miles per hour?

Dividing a Fraction by a Fraction

1. How many strips of cotton material $\frac{1}{8}$ yard wide can Mary get from a piece of material $\frac{3}{4}$ yard wide?



Mary pretended that each inch on her ruler stood for 1 yard. Then she used her ruler to find the answer.

From the ruler, you can see that Mary can get _?_ strips each $\frac{1}{8}$ yard wide.

Ann said, "If I wanted to find how many $\frac{1}{8}$'s there are in $\frac{3}{4}$, I would divide to find the answer. When we divided a fraction by a whole number, we wrote the whole number as a fraction, then used the rule for dividing by a fraction. This should be easier, because the fraction is already there. We could write it this way: $\frac{3}{4} \div \frac{1}{8} = \frac{3}{\cancel{4}_{1}} \times \frac{\overset{2}{\cancel{8}}}{1} = 6$. There are 6 one-eighths in $\frac{3}{4}$."

2. Check the answer to problem 1. Does $6 \times \frac{1}{8} = \frac{3}{4}$?

Now turn the page.

3. How many $\frac{3}{8}$'s of an inch are there in $\frac{3}{4}$ of an inch?
   (a) Use your ruler or draw a diagram to find the answer.
   (b) Find the answer by completing the following example: $\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3} =$?.
   (c) Check by putting your answer in the blank, then multiplying the following: ? $\times \frac{3}{8} = \frac{3}{4}$.

Copy and complete these examples and checks.

4. $\frac{4}{9} \div \frac{2}{9} = \frac{4}{9} \times \frac{9}{2} =$? ? $\times \frac{2}{9} = \frac{4}{9}$

5. $\frac{2}{3} \div \frac{2}{5} = \frac{2}{3} \times \frac{5}{2} =$? ? $\times \frac{2}{5} = \frac{2}{3}$

6. $\frac{1}{8} \div \frac{1}{2} = \frac{1}{8} \times \frac{2}{1} =$? ? $\times \frac{1}{2} = \frac{1}{8}$

7. $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1} =$? ? $\times \frac{1}{6} = \frac{1}{2}$

8. When a fraction is divided by a fraction, may the answer be a whole number? May it be a mixed number? May it be a proper fraction? In examples 4–7, was each answer larger than the dividend?

9. Make up a rule for dividing a fraction by a fraction.

Now study examples 4–7 and see if you can find a way to tell *about* how large the answer should be.

It may help you to write the division examples in the other form, for example 4, $\frac{2}{9}\overline{)\frac{4}{9}}$. The dividend is larger than the divisor. When you divide a larger number by a smaller number, the answer is ? than 1. In example 5, which is the larger number? Then the answer will be ? than 1. In example 6, which is the larger number? Then the answer will be ? than 1, in other words, a proper fraction. What can you say about example 7?

When you have the answer, what does it tell you?

In example 4, $\frac{4}{9} \div \frac{2}{9} = 2$. Can this mean that $\frac{4}{9}$ is twice as much as $\frac{2}{9}$? In example 5, $\frac{2}{3} \div \frac{2}{5} = \frac{5}{3} = 1\frac{2}{3}$. Can this mean that $\frac{2}{3}$ is $1\frac{2}{3}$ times as much as $\frac{2}{5}$? Can it mean that there is 1 whole two-fifths and $\frac{2}{3}$ of another two-fifths in two-thirds?

How would you interpret the answers to examples 6 and 7?

Use your rule to do the following divisions. In each one, decide whether your answer is sensible. Check each answer.

10.	$\frac{3}{15} \div \frac{3}{10}$	$\frac{2}{3} \div \frac{1}{6}$	$\frac{5}{8} \div \frac{3}{4}$	$\frac{3}{4} \div \frac{1}{4}$
11.	$\frac{2}{3} \div \frac{2}{9}$	$\frac{1}{2} \div \frac{7}{10}$	$\frac{1}{3} \div \frac{1}{4}$	$\frac{3}{4} \div \frac{1}{8}$
12.	$\frac{6}{8} \div \frac{1}{4}$	$\frac{1}{2} \div \frac{1}{4}$	$\frac{1}{2} \div \frac{2}{3}$	$\frac{1}{2} \div \frac{3}{4}$
13.	$\frac{3}{5} \div \frac{3}{10}$	$\frac{2}{5} \div \frac{1}{10}$	$\frac{2}{5} \div \frac{7}{10}$	$\frac{5}{8} \div \frac{5}{16}$
14.	$\frac{1}{3} \div \frac{7}{9}$	$\frac{1}{2} \div \frac{1}{6}$	$\frac{1}{4} \div \frac{5}{16}$	$\frac{5}{16} \div \frac{3}{4}$
15.	$\frac{1}{4} \div \frac{1}{8}$	$\frac{3}{4} \div \frac{1}{16}$	$\frac{1}{5} \div \frac{1}{4}$	$\frac{1}{5} \div \frac{3}{4}$

Do you think your rule for dividing a fraction by a fraction will work if the dividend or the divisor or both are improper fractions? Work each of the following examples using your rule. Compare each answer with the answer you can get by using the diagram beside the example.

16. $\frac{4}{2} \div \frac{1}{2}$

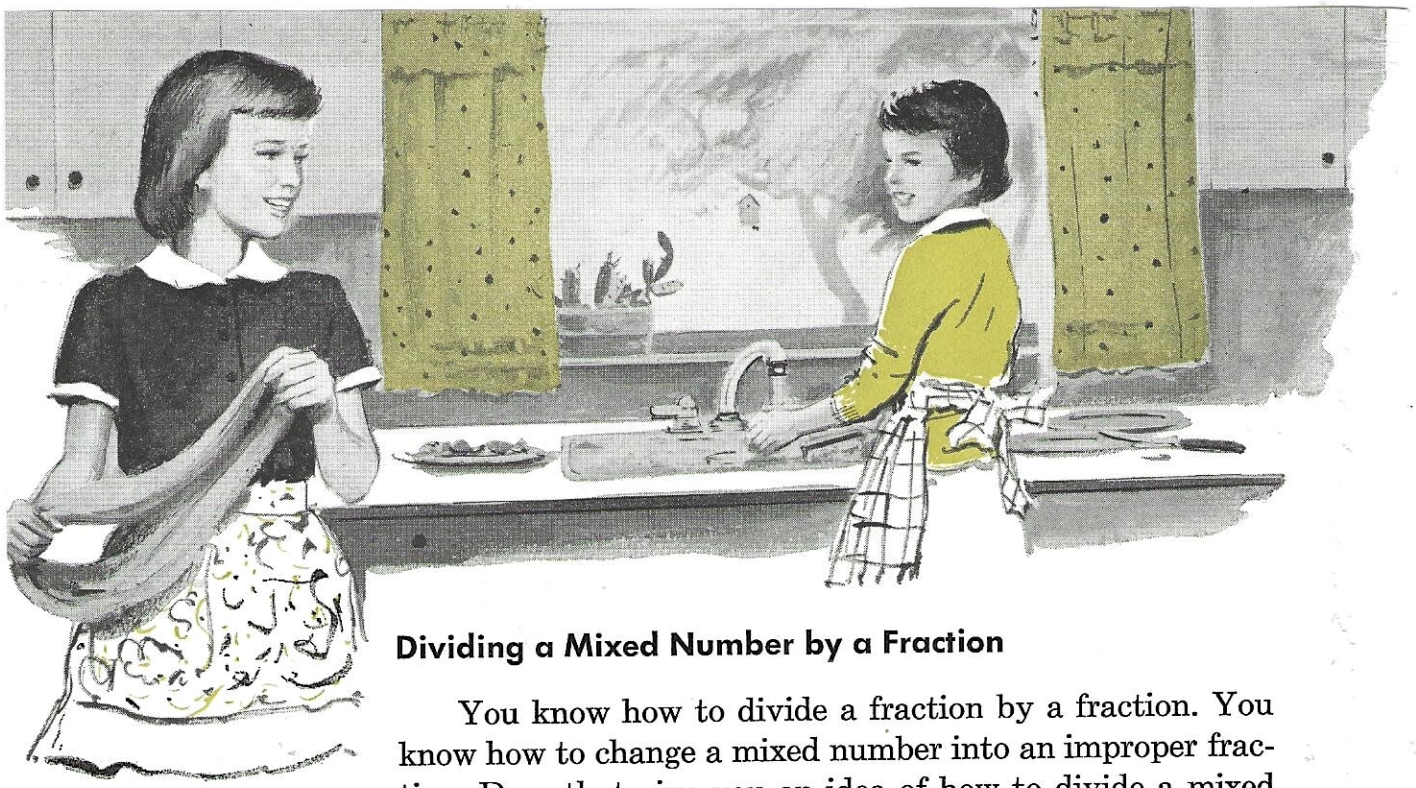
17. $\frac{9}{4} \div \frac{3}{4}$

18. $\frac{9}{2} \div \frac{3}{2}$

To divide by a fraction, multiply by the reciprocal of the divisor.

Copy, divide, and check.

19.	$\frac{7}{10} \div \frac{1}{10}$	$\frac{2}{3} \div \frac{1}{6}$	$\frac{3}{4} \div \frac{1}{16}$	$\frac{5}{16} \div \frac{3}{4}$
20.	$\frac{10}{9} \div \frac{5}{9}$	$\frac{5}{4} \div \frac{1}{8}$	$\frac{21}{16} \div \frac{3}{4}$	$\frac{7}{8} \div \frac{7}{10}$
21.	$\frac{1}{4} \div \frac{5}{4}$	$\frac{2}{5} \div \frac{11}{10}$	$\frac{5}{8} \div \frac{5}{2}$	$\frac{1}{3} \div \frac{7}{6}$
22.	$\frac{12}{5} \div \frac{6}{5}$	$\frac{9}{4} \div \frac{3}{2}$	$\frac{5}{2} \div \frac{10}{3}$	$\frac{11}{7} \div \frac{11}{7}$



Dividing a Mixed Number by a Fraction

You know how to divide a fraction by a fraction. You know how to change a mixed number into an improper fraction. Does that give you an idea of how to divide a mixed number by a fraction? See if you can solve the following problems without help.

1. The Morris girls like to make candy which they sometimes sell. They have made $4\frac{1}{2}$ pounds of candy. They plan to put $\frac{1}{4}$ pound of candy in each bag. How many bags of candy can they fill? $4\frac{1}{2} \div \frac{1}{4} = \frac{?}{2} \div \frac{1}{4} = \underline{?}$.

   Is your answer sensible? Should the answer be larger than $4\frac{1}{2}$? Will 18 bags of candy holding $\frac{1}{4}$ pound each hold $4\frac{1}{2}$ pounds of candy in all? $18 \times \frac{1}{4} = \underline{?}$.

2. It takes $\frac{1}{4}$ pound of butter to make one batch of chocolate taffy. How many batches can the girls make from $1\frac{1}{2}$ pounds of butter? Is your answer sensible? Check it.

Copy and complete these examples. Check each answer.

3. $3\frac{1}{3} \div \frac{2}{3} = \frac{10}{3} \div \frac{2}{3} = \frac{10}{3} \times \frac{3}{2} = \underline{?}$ $\qquad \underline{?} \times \frac{2}{3} = 3\frac{1}{3}$

4. $3\frac{3}{4} \div \frac{5}{6} = \frac{15}{4} \div \frac{5}{6} = \frac{15}{4} \times \frac{6}{5} = \underline{?}$ $\qquad \underline{?} \times \underline{?} = 3\frac{3}{4}$

5. $2\frac{2}{3} \div \frac{1}{6} = \frac{8}{3} \div \frac{1}{6} = \underline{?}$ $\qquad \underline{?} \times \underline{?} = \underline{?}$

6. Study the examples you just completed. When a mixed number is divided by a proper fraction, may the answer be a whole number? May it be a mixed number? Is the answer always larger than the dividend? When a mixed number is divided by a proper fraction, are you always dividing a larger number by a smaller number? May the answer be a proper fraction?

7. Make up a rule for dividing a mixed number by a fraction.

Use your rule to do the following divisions. In each one, decide whether your answer is sensible. Then check each answer by multiplying.

8.	$2\frac{1}{2} \div \frac{1}{4}$	$4\frac{1}{3} \div \frac{1}{6}$	$2\frac{1}{4} \div \frac{1}{2}$	$5\frac{1}{2} \div \frac{3}{4}$
9.	$3\frac{1}{2} \div \frac{3}{8}$	$2\frac{1}{3} \div \frac{2}{9}$	$1\frac{1}{4} \div \frac{1}{8}$	$4\frac{2}{3} \div \frac{1}{9}$
10.	$4\frac{1}{4} \div \frac{1}{2}$	$2\frac{1}{5} \div \frac{1}{5}$	$2\frac{1}{5} \div \frac{1}{10}$	$1\frac{3}{10} \div \frac{5}{6}$

Dividing a Fraction by a Mixed Number

1. John knows that it is $\frac{1}{4}$ mile from his house to the post office. One day he timed himself as he walked this distance. He found it took him $3\frac{3}{4}$ minutes to walk the $\frac{1}{4}$ mile. He wondered what part of a mile he could walk in 1 minute.

To find the answer, John divided $\frac{1}{4}$ by $3\frac{3}{4}$. He changed $3\frac{3}{4}$ to an improper fraction. Then he multiplied by the reciprocal of the divisor.

John's example and the check are partly shown below. Complete each of them. What part of a mile could John walk in 1 minute?

$\frac{1}{4} \div 3\frac{3}{4} = \frac{1}{4} \div \frac{15}{4} = \frac{1}{4} \times \frac{4}{15} =$? ? $\times 3\frac{3}{4} = \frac{1}{4}$

Now turn the page.

Copy and complete the examples below. Check each answer.

2. $\frac{2}{3} \div 1\frac{1}{3} = \frac{2}{3} \div \frac{4}{3} = \frac{2}{3} \times \frac{3}{4} =$? ? $\times 1\frac{1}{3} = \frac{2}{3}$

3. $\frac{1}{2} \div 2\frac{1}{4} = \frac{1}{2} \div \frac{9}{4} =$? ? $\times$? $= \frac{1}{2}$

4. $\frac{3}{10} \div 2\frac{2}{5} = \frac{3}{10} \div \frac{12}{5} =$? ? $\times$? $=$?

5. When a proper fraction is divided by a mixed number, is the dividend less than 1? Is the divisor more than 1? Must the answer always be a proper fraction? Must the answer be smaller than the dividend?

6. Make up a rule for dividing a fraction by a mixed number.

Use your rule to do the following divisions. In each one, decide whether your answer is sensible. Check each answer.

7.	$\frac{2}{3} \div 1\frac{1}{3}$	$\frac{1}{2} \div 2\frac{1}{2}$	$\frac{5}{6} \div 2\frac{1}{6}$	$\frac{5}{6} \div 3\frac{1}{3}$
8.	$\frac{3}{8} \div 2\frac{1}{4}$	$\frac{5}{8} \div 2\frac{1}{2}$	$\frac{3}{10} \div 1\frac{1}{4}$	$\frac{3}{4} \div 1\frac{1}{2}$
9.	$\frac{7}{10} \div 4\frac{1}{5}$	$\frac{5}{12} \div 1\frac{1}{4}$	$\frac{7}{8} \div 2\frac{1}{2}$	$\frac{3}{10} \div 2\frac{1}{5}$

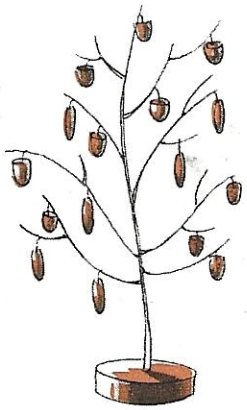
10. Mrs. Perry has been dieting to lose weight. She lost $\frac{3}{4}$ of a pound in $4\frac{1}{2}$ days. What was her average daily loss?

No Pencils, Please

Compare the size of the dividend with the size of the divisor in each example below. Then tell whether the answer will be 1, more than 1, or less than 1.

1.	$\frac{3}{4} \div \frac{3}{4}$	$\frac{3}{4} \div \frac{1}{4}$	$\frac{3}{4} \div \frac{1}{2}$	$\frac{3}{4} \div 2\frac{1}{4}$
2.	$\frac{2}{3} \div \frac{1}{9}$	$\frac{7}{8} \div 3\frac{1}{5}$	$\frac{2}{5} \div \frac{9}{10}$	$1\frac{1}{6} \div 1\frac{1}{6}$
3.	$\frac{1}{2} \div 4$	$4\frac{1}{2} \div 5$	$3 \div \frac{2}{7}$	$8\frac{1}{2} \div 8$

Dividing a Mixed Number by a Mixed Number



1. Joan is fixing gumdrops to hang on a sugar-plum tree. She needs $1\frac{1}{2}$ inches of wire to attach to each gumdrop. How many gumdrops can she fix with $7\frac{1}{2}$ inches of wire?

To find the answer, Joan divided $7\frac{1}{2}$ by $1\frac{1}{2}$. First she changed the mixed numbers to improper fractions. Then she multiplied by the reciprocal of the divisor. Joan's example and the check are partly shown below. Complete each of them. How many gumdrops could Joan fix?

$7\frac{1}{2} \div 1\frac{1}{2} = \frac{15}{2} \div \frac{3}{2} = \frac{15}{2} \times \frac{2}{3} =$? ? $\times 1\frac{1}{2} = 7\frac{1}{2}$

Copy and complete the examples below. Check each answer.

2. $4\frac{1}{2} \div 1\frac{1}{2} = \frac{9}{2} \div \frac{3}{2} = \frac{9}{2} \times \frac{2}{3} =$? ? $\times 1\frac{1}{2} = 4\frac{1}{2}$
3. $5\frac{2}{3} \div 2\frac{1}{3} = \frac{17}{3} \div \frac{7}{3} =$? ? $\times$? $= 5\frac{2}{3}$
4. $1\frac{3}{4} \div 2\frac{4}{5} = \frac{7}{4} \div \frac{14}{5} =$? ? $\times$? $=$?
5. When a mixed number is divided by a mixed number, may the answer be a whole number? May it be a mixed number? May it be a proper fraction? Is the answer always smaller than the dividend? How can you tell before you divide *about* how large your answer should be?
6. Make up a rule for dividing a mixed number by a mixed number.

Use your rule to do the following divisions. In each one, decide whether your answer is sensible. Check all your answers.

7.	$4\frac{1}{3} \div 1\frac{1}{3}$	$7\frac{1}{2} \div 1\frac{1}{2}$	$3\frac{3}{4} \div 2\frac{1}{2}$	$4\frac{5}{6} \div 3\frac{1}{3}$
8.	$1\frac{3}{8} \div 2\frac{1}{8}$	$1\frac{2}{3} \div 3\frac{1}{2}$	$1\frac{3}{4} \div 2\frac{5}{8}$	$3\frac{2}{3} \div 4\frac{5}{6}$
9.	$8\frac{1}{3} \div 2\frac{3}{4}$	$3\frac{1}{10} \div 2\frac{2}{3}$	$2\frac{4}{5} \div 3\frac{2}{3}$	$4\frac{1}{3} \div 6\frac{3}{4}$

Practice What You Have Learned

1. Write a division example that tells how many badges each $\frac{1}{4}$ yard long you can get from 3 yards of ribbon. Is the answer more than 3?

2. When a whole number is divided by a proper fraction, the answer is always _?_ than the whole number.

3. In the division $\frac{4}{5} \div \frac{2}{3}$, which number is the dividend? Which number is the divisor? Of which number do you use the reciprocal in multiplying to find the answer?

4. See if you can answer the following questions without completing the divisions. Complete them if you need to. Which of the examples below shows that when a fraction is divided by a fraction, the answer can be a whole number? A mixed number? A proper fraction?

   (a) $\frac{1}{5} \div \frac{1}{2}$ (b) $\frac{3}{4} \div \frac{1}{2}$ (c) $\frac{3}{5} \div \frac{1}{5}$

5. Write a division example that tells how many pennants each $1\frac{1}{2}$ yards long you can get from 6 yards of cloth. Is the answer less than 6?

6. When any number is divided by a mixed number, the answer is always _?_ than the dividend.

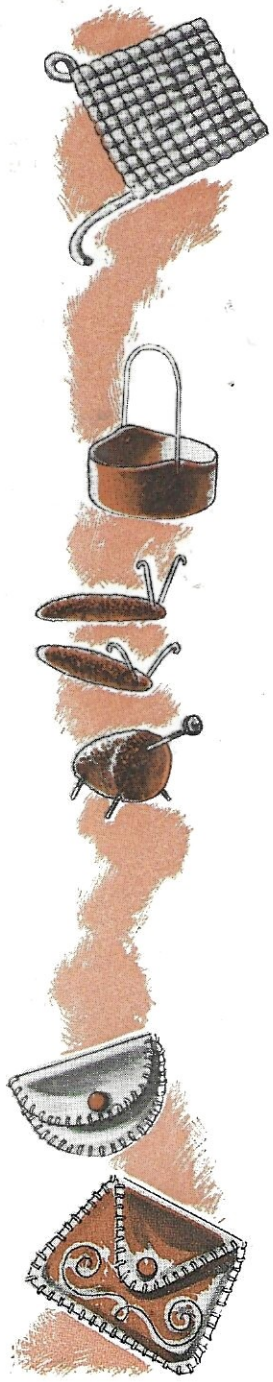
Copy and divide. Use any correct method you like. Be sure that each of your answers is sensible. Check the first example in each row.

7.	$8 \div \frac{1}{4}$	$9 \div \frac{1}{2}$	$6 \div \frac{1}{5}$	$5 \div \frac{1}{3}$
8.	$2 \div \frac{1}{6}$	$4 \div \frac{1}{2}$	$3 \div \frac{1}{3}$	$2 \div \frac{1}{8}$
9.	$3 \div \frac{3}{4}$	$8 \div \frac{4}{5}$	$4 \div \frac{2}{3}$	$5 \div \frac{5}{8}$
10.	$6 \div \frac{2}{3}$	$5 \div \frac{5}{6}$	$2 \div \frac{8}{9}$	$3 \div \frac{3}{5}$
11.	$\frac{1}{2} \div 4$	$\frac{1}{8} \div 2$	$\frac{1}{7} \div 3$	$\frac{1}{6} \div 5$

12. $\frac{1}{4} \div 3$	$\frac{1}{3} \div 2$	$\frac{1}{4} \div 2$	$\frac{1}{8} \div 4$
13. $\frac{3}{4} \div 6$	$\frac{2}{5} \div 4$	$\frac{2}{3} \div 8$	$\frac{3}{5} \div 15$
14. $\frac{7}{8} \div 7$	$\frac{5}{6} \div 10$	$\frac{4}{5} \div 8$	$\frac{3}{4} \div 12$
15. $2\frac{1}{2} \div 5$	$3\frac{1}{5} \div 2$	$4\frac{2}{3} \div 7$	$6\frac{3}{4} \div 9$
16. $6\frac{1}{2} \div 4$	$4\frac{1}{2} \div 3$	$5\frac{1}{3} \div 4$	$1\frac{1}{2} \div 3$
17. $\frac{2}{3} \div \frac{1}{2}$	$\frac{5}{6} \div \frac{1}{3}$	$\frac{3}{4} \div \frac{1}{2}$	$\frac{3}{4} \div \frac{3}{8}$
18. $\frac{1}{5} \div \frac{1}{2}$	$\frac{3}{5} \div \frac{1}{5}$	$\frac{7}{8} \div \frac{2}{3}$	$\frac{2}{3} \div \frac{2}{5}$
19. $3\frac{1}{2} \div \frac{3}{4}$	$8\frac{1}{4} \div \frac{3}{8}$	$4\frac{1}{3} \div \frac{5}{6}$	$5\frac{2}{5} \div \frac{1}{5}$
20. $1\frac{1}{2} \div \frac{1}{4}$	$6\frac{3}{4} \div \frac{9}{10}$	$7\frac{1}{2} \div \frac{2}{3}$	$2\frac{1}{8} \div \frac{8}{9}$
21. $\frac{3}{4} \div 1\frac{1}{2}$	$\frac{7}{8} \div 1\frac{1}{4}$	$\frac{4}{5} \div 1\frac{1}{5}$	$\frac{5}{6} \div 2\frac{1}{2}$
22. $\frac{7}{16} \div 4\frac{3}{4}$	$\frac{1}{2} \div 3\frac{1}{2}$	$\frac{3}{4} \div 2\frac{3}{8}$	$\frac{3}{16} \div 1\frac{3}{4}$
23. $5\frac{1}{2} \div 1\frac{1}{2}$	$4\frac{3}{4} \div 2\frac{1}{4}$	$2\frac{1}{8} \div 1\frac{1}{2}$	$6\frac{2}{3} \div 1\frac{4}{5}$
24. $1\frac{1}{2} \div 3\frac{1}{2}$	$7\frac{1}{2} \div 6\frac{2}{3}$	$1\frac{1}{4} \div 6\frac{3}{4}$	$7\frac{1}{2} \div 1\frac{1}{2}$

Just for Fun

1. In an Eastern city, the boys in the Little League collected 3 miles of pennies to give to the Jimmy Fund. Just for fun, guess how much money that would be. Then, with your ruler, measure a penny across its widest part. See if you can find out how much 3 miles of pennies are worth. How close was your guess?
2. If it takes 144 pennies to weigh a pound, how much would 3 miles of pennies weigh? Is that more than a ton?
3. Tom said that 3 miles of quarters would be worth 25 times as much as 3 miles of pennies. Is that true? Why or why not?



Problems Using Fractions

1. June needs $1\frac{1}{2}$ yards of clothesline for making a hot mat. How many hot mats can she make from 3 yards of clothesline?
2. To help him to make heads from old light bulbs, plaster of Paris, and paper, Jim needed strips of paper $\frac{1}{4}$ inch wide. How many strips of paper can Jim get from a piece 9 inches wide?
3. For each candy dish that Ann is making, she needs a piece of wire $8\frac{1}{2}$ inches long for a handle. How many handles can Ann make from a piece of wire 17 inches long?
4. Bob is making gumdrop creatures. For each caterpillar that he wants to make he needs a piece of wire $1\frac{1}{2}$ inches long to use for antennae. How many caterpillars can Bob put antennae on if he has a piece of wire 6 inches long?
5. It takes Joe an average of $\frac{3}{4}$ hour to cut a necktie holder with his jig saw. How many necktie holders can he cut in three hours?
6. Judy has $\frac{1}{2}$ yard of cord to use for the 2 arms on the spool doll she is making. How much cord can she use for each arm?
7. Alice has $\frac{2}{3}$ of a yard of narrow ribbon to use to tie onto 2 fruit favors. How much ribbon can she use on each favor?
8. Phil has $7\frac{1}{2}$ yards of plastic lacing to use for lacing coin purses. He has 5 purses to lace. How much lacing can he use on each purse?
9. In making cookies Mary wants to add $\frac{3}{4}$ cup of sweet milk in two equal parts. How much milk will she add each time?

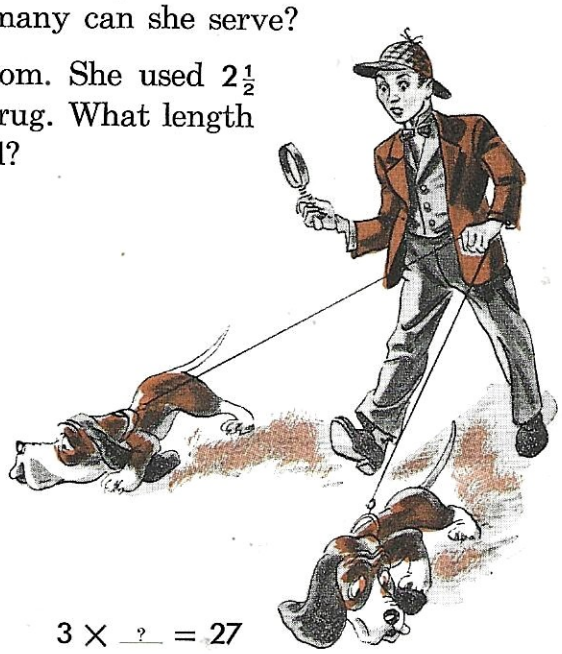
10. The recipe that Marion is using to make cookies tells her to add $3\frac{1}{2}$ cups of flour in 4 equal parts. How much flour will she add each time?

11. How many pieces of wire $\frac{3}{8}$ yard long can Jerry cut from a piece of wire $\frac{3}{4}$ yard long?

12. How many pennants each $2\frac{1}{4}$ feet long can be cut from a strip of felt $6\frac{3}{4}$ feet long?

13. Mrs. Ames bought a turkey that weighed $10\frac{1}{2}$ pounds. If she allows $\frac{3}{4}$ pound to each person, how many can she serve?

14. Kay is knitting a rug for her bedroom. She used $2\frac{1}{2}$ pounds of wool to knit $\frac{3}{4}$ yard of the rug. What length could she knit with one pound of wool?



Find the Missing Number

There are three types of problems using multiplication and division of fractions that will be very easy for you to solve if you think about a few basic ideas that you already know.

1. How do you find the missing number in each example below?

$3 \times 9 = \_?\_$	$\_?\_ \times 9 = 27$	$3 \times \_?\_ = 27$
$12 \times 15 = \_?\_$	$\_?\_ \times 21 = 126$	$42 \times \_?\_ = 210$

2. Could you use the same methods if one or more of the numbers in the examples were fractions?

3. See if you can find the missing number in each example below.

$\frac{1}{2} \times 8 = \_?\_$	$\_?\_ \times 8 = 4$	$\frac{1}{2} \times \_?\_ = 4$
$\frac{3}{4} \times 20 = \_?\_$	$\_?\_ \times 12 = 8$	$\frac{1}{2} \times \_?\_ = 24$

Now turn the page.

4. Are the following statements true?
   (a) Every complete multiplication involves three numbers.
   (b) Multiplier × multiplicand = product.
   (c) If you know any two of the numbers, you can find the third number.
   (d) To find a missing product, you multiply.
   (e) To find a missing multiplier or a missing multiplicand, you divide the product by the other number you know.
   (f) When you divide a number by a larger number, the answer is a proper fraction.

5. From the words in each set of parentheses, choose the word that will make the following statements true.
   (a) To find the missing number in $\frac{3}{4} \times 16 = \_?\_$, you (multiply, divide) 16 by $\frac{3}{4}$. The answer will be (larger, smaller) than 16.
   (b) To find the missing number in $\_?\_ \times 12 = 4$, you (multiply, divide) 4 by 12. The answer will be (more, less) than 1.
   (c) To find the missing number in $\frac{1}{4} \times \_?\_ = 5$, you (multiply, divide) 5 by $\frac{1}{4}$. The answer will be (larger, smaller) than 5.

6. Why can you find no *one* correct answer to this? $\_?\_ \times 0 = 0$.

Finding a Part of a Number

1. Tom had 36 marbles. He says he lost $\frac{3}{4}$ of them. How many did he lose? (Remember: $\frac{3}{4}$ of $36 = \frac{3}{4} \times 36$.)

2. How many ounces in a pound. How many ounces in $\frac{1}{2}$ pound? In $\frac{1}{4}$ pound? In $\frac{3}{4}$ pound? In $\frac{5}{8}$ pound? In $\frac{2}{3}$ pound?

3. How many inches in a foot? How many inches in $\frac{1}{2}$ foot? In $\frac{1}{4}$ foot? In $\frac{3}{4}$ foot? In $\frac{5}{6}$ foot? In $\frac{2}{3}$ foot?

4. Mary received \$5.00 for her birthday. She spent $\frac{3}{10}$ of it. How much did she spend?

5. Mr. Johnson's fuel oil tank holds 300 gallons when full. It is now $\frac{3}{4}$ full. How many gallons of oil are in the tank?

Finding What Part One Number Is of Another

1. Mrs. Jones baked 24 cookies. Her boys ate 6 of them. What part of the cookies did the boys eat?

Are "What part of 24 is 6?" and "6 is what part of 24?" two ways of asking the same question? Is that the question this problem asks?

Two ways to think about this question are shown below. Study them both; then choose the way that seems better to you.

First way: Find the ratio of 6 to 24 and express the ratio as a fraction in lowest terms. $\frac{6}{24} = \_?\_$.

Second way: Write the question in symbols like this: $\_?\_ \times 24 = 6$. Now you can see that you know the product of two numbers and one of the numbers. Divide to find the other number. $6 \div 24 = \frac{6}{24} = \_?\_$.

Should the answer be a proper fraction? Are 6 cookies $\frac{1}{4}$ of 24 cookies?

2. Jim worked 20 minutes weeding the garden. What part of an hour did he work?

3. Mr. Johnson bought a new 300-gallon tank for his fuel oil. If he puts in 200 gallons of oil, what fraction of the tank will be filled?

4. 9 is _?_ of 27 — 7 is _?_ of 56 — 10 is _?_ of 20 — 50 is _?_ of 200

5. _?_ of 64 is 8 — _?_ of 18 is 9 — _?_ of 25 is 15 — _?_ of 50 is 35

6. 8 is _?_ of 800 — 12 is _?_ of 72 — _?_ of 100 is 5 — _?_ of 36 is 27

7. A bale of cotton weighs about 500 pounds. A man who harvests 300 pounds per acre is getting about _?_ of a bale per acre.

8. Jim bought a bicycle for $48.00. Two years later he sold it for $30.00. He sold it for _?_ of what it cost him.

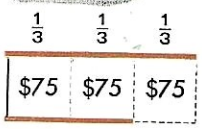
9. Mary weighs 80 pounds, Carl weighs 84 pounds, Tony weighs 96 pounds, and Linda weighs 72 pounds. Linda weighs _?_ as much as Mary. Carl weighs _?_ as much as Tony. Mary weighs _?_ as much as Tony.

Finding the Whole When You Know a Part

1. Read the sign in the picture at the left. Roger's father paid \$150 for the television set he bought at the sale. How much was the original price of this set?

The problem tells you that $\frac{2}{3}$ of the original price is \$150. It asks you to find the whole price when you know $\frac{2}{3}$ of it.

Three ways to think about the problem are shown below. Study them all, then choose the way that seems best to you.



\$150 = $\frac{2}{3}$ price

First way: Draw a picture somewhat like the one at the left. Use the picture to find the answer. Why do you divide \$150 by 2 to find the number to put in each third in the drawing? Why must the answer be more than \$150? Is your answer correct? Is \$150 equal to $\frac{2}{3}$ of \$225?

Second way: If $\frac{2}{3}$ of the original price is \$150, then $\frac{1}{3}$ of the original price is \$75. All $(\frac{3}{3})$ of the original price would be 3 times as much as $\frac{1}{3}$ of it. $3 \times \$75 =$ ___?___.

Third way: Write in symbols "$\frac{2}{3}$ of the original price is \$150." Use a ___?___ for the number you want to find. $\frac{2}{3} \times$ ___?___ = \$150. Now you can see that you know the product of two numbers and one of the numbers. Divide to find the other number. $\$150 \div \frac{2}{3} =$ ___?___.

2. $\frac{2}{3}$ of a number is 20. What is the number?

3. $\frac{8}{9}$ of a number is 16. What is the number?

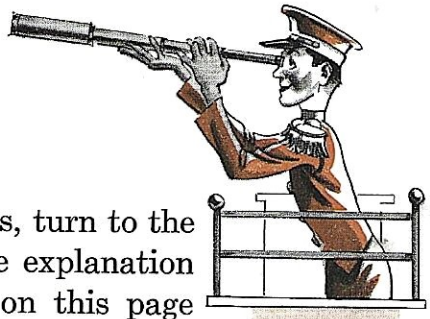
4. $\frac{1}{5}$ of a number is 7. What is the number?

5. When the gasoline tank on our car is $\frac{3}{4}$ full, we have 12 gallons of gasoline. How many gallons does the tank hold?

6. Henry says $\frac{2}{5}$ of all his chicks are Leghorns. He has 40 Leghorns. How many chicks in all does he have?

Checking Up on Division of Fractions

DIAGNOSTIC TEST ON PAGES 92, 227, AND 253–278



If you make mistakes in a row of examples, turn to the page shown at the end of the row. Study the explanation and repeat the practice. Then try the row on this page again.

Change the following mixed numbers to improper fractions.

1.	$2\frac{1}{8}$	$1\frac{2}{3}$	$2\frac{4}{5}$	$7\frac{1}{2}$	$3\frac{3}{10}$	$2\frac{4}{7}$	$7\frac{2}{9}$	$4\frac{3}{16}$	227

Change the following improper fractions to whole or mixed numbers.

2.	$\frac{15}{6}$	$\frac{10}{4}$	$\frac{21}{8}$	$\frac{24}{2}$	$\frac{32}{9}$	$\frac{40}{5}$	$\frac{15}{15}$	$\frac{35}{15}$	92

3. What is the divisor in $3\frac{1}{2} \div \frac{5}{8}$? 253

Copy and divide.

4.	$\frac{1}{3} \div 2$	$\frac{2}{5} \div 4$	$\frac{5}{7} \div 10$	$\frac{7}{8} \div 14$	263
5.	$7 \div \frac{1}{3}$	$3 \div \frac{5}{8}$	$8 \div \frac{4}{4}$	$9 \div \frac{6}{7}$	256
6.	$\frac{3}{8} \div \frac{3}{4}$	$\frac{2}{5} \div \frac{3}{10}$	$\frac{3}{4} \div \frac{5}{8}$	$\frac{7}{12} \div \frac{1}{6}$	265
7.	$6 \div 1\frac{1}{2}$	$4 \div 2\frac{2}{3}$	$2 \div 3\frac{1}{3}$	$8 \div 2\frac{1}{2}$	257
8.	$4\frac{1}{2} \div 4$	$2\frac{1}{4} \div 3$	$4\frac{3}{8} \div 10$	$8\frac{1}{3} \div 5$	264
9.	$2\frac{1}{4} \div \frac{1}{2}$	$2\frac{2}{3} \div \frac{5}{8}$	$4\frac{2}{5} \div \frac{7}{10}$	$1\frac{5}{6} \div \frac{3}{8}$	268
10.	$\frac{4}{5} \div 1\frac{1}{3}$	$\frac{3}{4} \div 2\frac{1}{4}$	$\frac{5}{6} \div 2\frac{1}{5}$	$\frac{7}{8} \div 1\frac{3}{4}$	269
11.	$7\frac{1}{3} \div 2\frac{1}{4}$	$1\frac{1}{8} \div 5\frac{5}{8}$	$4\frac{4}{5} \div 6\frac{1}{5}$	$4\frac{5}{6} \div 3\frac{1}{3}$	271

12. What number is $\frac{2}{3}$ of 24? 276

13. What part of 15 is 5? 277

14. 14 is $\frac{1}{2}$ of what number? 278

What Have You Learned about Division of Fractions?

CHAPTER TEST ON MEANINGS AND SKILLS

1. What part of 16 is 8? Express as a fraction in lowest terms.
2. Before dividing a mixed number, change the mixed number to an _?_.
3. To divide by a fraction, multiply by its _?_.
4. The reciprocal of $\frac{4}{7}$ is _?_.
5. The answer to $5\frac{1}{3} \div 2\frac{1}{4}$ should be more than _?_ and less than _?_.
6. $1 = \frac{?}{4}$ $\quad 2 = \frac{?}{4}$ $\quad 3 = \frac{?}{5}$ $\quad 6 = \frac{?}{3}$ $\quad 5 = \frac{?}{2}$ $\quad 8 = \frac{?}{6}$

Copy and divide. Be especially careful to copy each example correctly.

7.	$\frac{1}{3} \div 4$	$\frac{3}{5} \div 6$	$\frac{5}{8} \div 10$	$\frac{3}{7} \div 12$
8.	$5 \div \frac{1}{2}$	$3 \div \frac{7}{8}$	$12 \div \frac{4}{5}$	$15 \div \frac{5}{6}$
9.	$\frac{5}{8} \div \frac{5}{6}$	$\frac{1}{5} \div \frac{3}{10}$	$\frac{3}{16} \div \frac{5}{8}$	$\frac{7}{12} \div \frac{3}{4}$
10.	$9 \div 1\frac{1}{2}$	$8 \div 2\frac{2}{3}$	$2 \div 5\frac{1}{3}$	$6 \div 2\frac{1}{2}$
11.	$2\frac{1}{4} \div 3$	$4\frac{2}{3} \div 7$	$3\frac{3}{8} \div 3$	$6\frac{2}{3} \div 5$
12.	$2\frac{2}{3} \div \frac{5}{8}$	$2\frac{3}{4} \div \frac{1}{2}$	$4\frac{4}{5} \div \frac{6}{7}$	$2\frac{1}{7} \div \frac{3}{8}$
13.	$\frac{4}{7} \div 1\frac{1}{3}$	$\frac{3}{5} \div 2\frac{1}{4}$	$\frac{5}{6} \div 3\frac{3}{5}$	$\frac{7}{8} \div 4\frac{1}{5}$
14.	$5\frac{1}{3} \div 2\frac{1}{4}$	$1\frac{1}{8} \div 3\frac{3}{5}$	$2\frac{1}{3} \div 1\frac{1}{3}$	$1\frac{3}{4} \div 2\frac{4}{5}$

What Have You Learned about Division of Fractions?

CHAPTER TEST ON PROBLEM SOLVING

1. Ruth and Sue made dresses for Dutch dolls. They shared equally $\frac{5}{6}$ yard of cloth for dresses for the 2 dolls. What part of a yard did they use for each doll?

2. Sandra and Louise made 6 quarts of fruit punch for a party. They served this in cups each holding $\frac{1}{8}$ quart. How many cups of fruit punch did they have?

3. How many strips of aluminum foil $\frac{1}{8}$ inch wide can Betty cut from a strip $\frac{3}{4}$ inch wide?

4. Mrs. Hart roasted a piece of meat that weighed $6\frac{3}{4}$ pounds. She allowed $\frac{3}{4}$ pound for each serving. How many servings can be made?

5. Donald has $2\frac{1}{2}$ pounds of carrots for his rabbits. He feeds them $\frac{1}{4}$ pound each day. How many days will these carrots last?

6. Doris is drawing patterns $2\frac{3}{4}$ inches wide in rows on a poster. How many rows of patterns can she fit into a space $16\frac{1}{2}$ inches wide?

7. Mr. Noble has a flower plot 10 feet wide. He divided the plot into strips $2\frac{1}{2}$ feet wide. How many strips did he have?

8. If a gallon of water weighs $8\frac{1}{3}$ pounds, how much does a quart of water weigh?

9. Ted caught 5 fish weighing a total of $12\frac{1}{2}$ pounds. What was the average weight of a fish?

10. Tom walked around the lake, a distance of $3\frac{1}{3}$ miles, in $1\frac{2}{3}$ hours. How many miles per hour did he average?

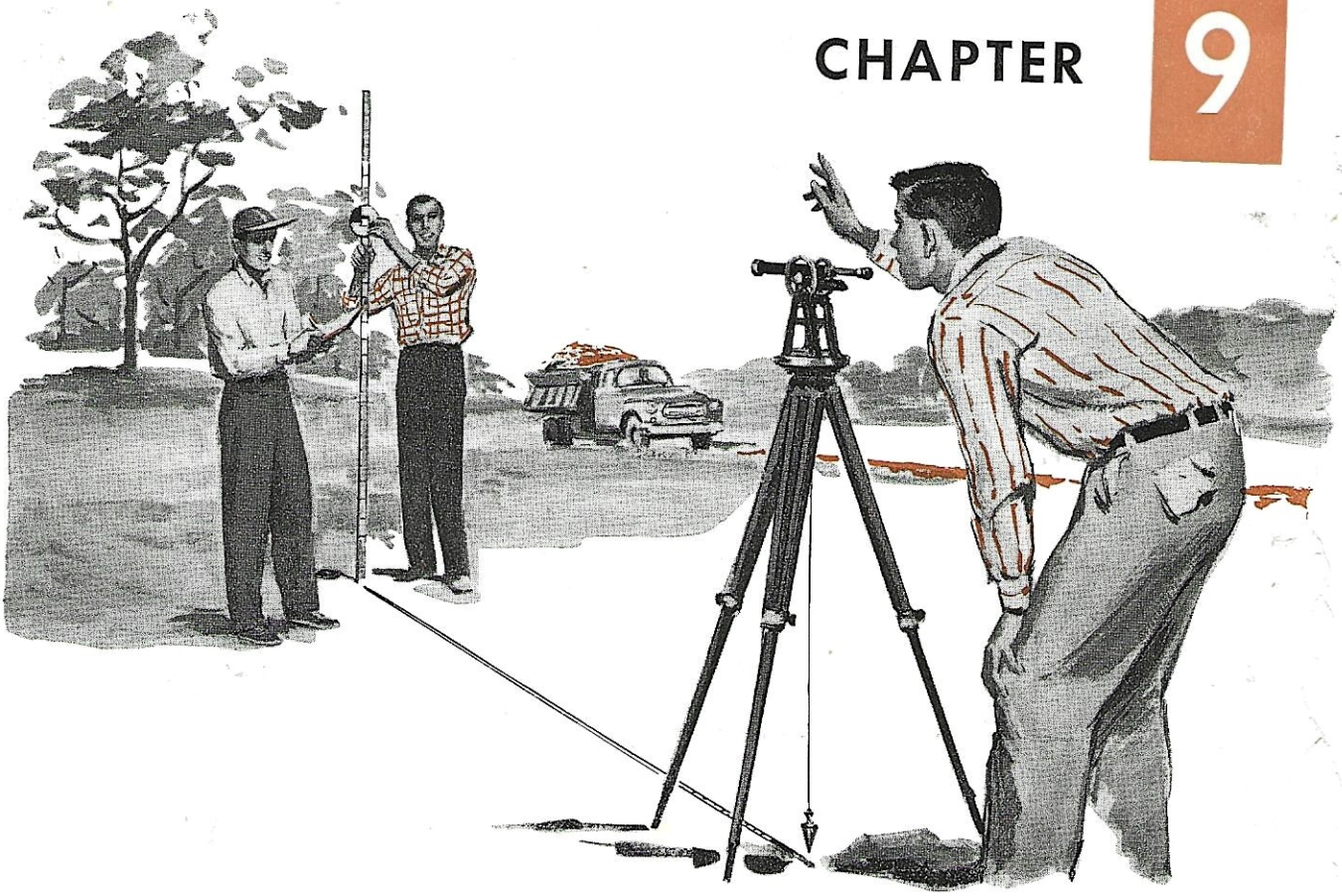
11. Jack has 6 feet of rope which he will cut into pieces $1\frac{1}{2}$ feet long. How many pieces of rope can he get?

12. Alice has 6 oranges left. These are $\frac{1}{4}$ the number sent to her by her cousin. How many oranges did her cousin send to her?

13. How many doughnuts would you get if you bought $\frac{3}{4}$ dozen doughnuts?

14. Some kinds of hot-dog rolls have 8 in a package. What part of a dozen is that?

CHAPTER 9

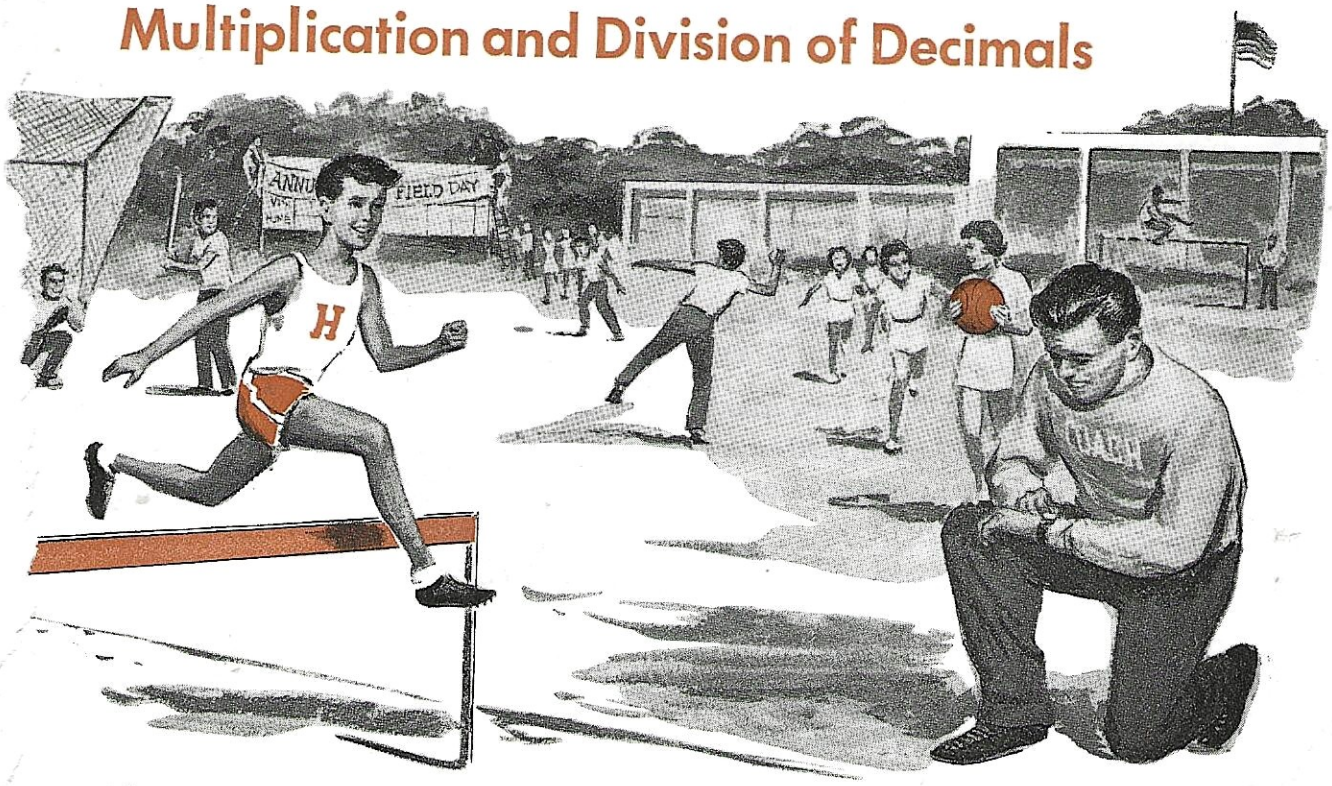


Fixing the Athletic Field

The surveyors and the other men are working on the school's new football field. They are using tapes and poles that measure lengths and heights in feet and tenths and hundredths of a foot. The surveyors will have to multiply and divide some of the numbers they get from these measurements to tell how much dirt must be put on the field to make it level.

The students are using the other part of the field to get ready for their annual field day. They will have to use decimals in figuring out batting averages in the baseball games, money matters, records for the races, and perhaps for the scores they will use in awarding the prizes.

Multiplication and Division of Decimals

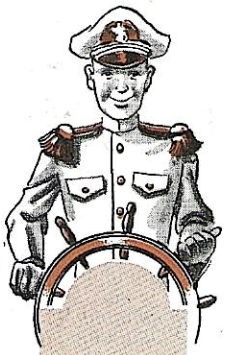


The Big Little Dot

Working with decimals is easy. You work with decimals just as you work with whole numbers except that you must know where to place the decimal point in your answers. This is a very important matter. Putting the decimal point in the wrong place is almost the worst mistake you can make.

1. A bookkeeper for a construction firm billed at \$106.20 an item which should have been \$10,620. The customer paid the \$106.20. The company discovered the mistake and tried to recover the difference. They found that they could not make the customer pay any more. How much money did the company lose because of this misplaced decimal point?

2. Suppose that a measurement correct to the nearest hundredth foot is 36.48 ft. Which is closer to the correct measurement, 41.02 ft. or 3.648 ft.?



Are You Ready for Multiplication and Division of Decimals?

READINESS TEST

If you can do the work on these pages correctly, you are ready for multiplication and division of decimals.

160 1. Tell the meaning and place value of each figure in 576.328. Begin: 5 means 5 __?__, and so on.

171 2. In $222.22, how many times as large is $200 as $20? $20 as $2? $2 as $.20? $.20 as $.02? Each place in our number system has a value __?__ times the value of the next place to its right.

158, 170 3. Write these common fractions as decimal fractions.

$\frac{2}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{100}$ $\frac{15}{100}$ $\frac{76}{100}$ $\frac{229}{1000}$ $\frac{15}{1000}$ $\frac{7}{1000}$

158, 170 4. Write these decimal fractions as common fractions.

.4 .05 .27 .8 .49 .008 .15 .058

179 5. Which is larger? .3 or .03 .5 or .005 .006 or .06
1.25 or .125 .035 or .35 .75 or 7.5

167 6. What common fraction of $1.00 is $.10? $.20? $.25? $.30? $.40? $.50? $.60? $.70? $.75? $.80?

Copy and add.

164 7.

.16	.07	.78	2.39	5.00
.38	.09	.69	4.68	3.90

Copy and subtract.

165 8.

8.67	9.63	.94	6.30	8.00
4.32	6.47	.38	4.20	3.54

Copy and multiply.

39 9.

$2.14	$3.04	$53.96	$1.30	$24.06
2	7	5	12	49

Copy and divide.

10. $3\overline{)\$12.69}$ $8\overline{)\$25.76}$ $9\overline{)\$20.70}$ $4\overline{)\$.20}$ $6\overline{)\$12.12}$ — 53

Round the following prices to the nearest dollar.

11. \$2.98 \$4.19 \$12.49 \$49.69 \$.95 — 12

Round the following prices to the nearest ten cents.

12. 12¢ 48¢ \$.89 \$.98 \$2.37 — 12

13. Multiply each of the following numbers by 10. Write only the answers. 7 34 256 80 910 — 40

14. Multiply each of the numbers in row 13 by 100. — 40

15. Multiply each of the numbers in row 13 by 1000. — 40

Write the division example that you would use to find the answer to each of the following questions. Work each example.

16. (a) How many 25-cent tickets can you buy for 75 cents? — 53
(b) How many 20-cent balloons can you get for a dollar?
(c) How many nickels can you get for \$2.00?

17. Phil's camera cost \$20.86. Alice's camera cost \$2.98. Round each of those prices to the nearest dollar. Phil's camera cost about _?_ times as much as Alice's. Divide \$20.86 by \$2.98. Did you get the answer you expected to get? — 53

Do as the signs tell you to do.

18. $3 \times 2\frac{1}{10}$ $\frac{3}{10} \times \frac{7}{10}$ $\frac{9}{10} \div \frac{3}{10}$ $\frac{35}{100} \div 5$ — 230, 263

19. Using decimals in place of the fractions, write each example in row 18 with its answer. — 158

If you made mistakes in any row, turn to the page listed beside the row. Study the explanation given there. Then work on the practice examples until you are sure you are ready for multiplication and division of decimals.

Rounding Decimals and Mixed Decimals

When Mr. Jones looked at the surveyor's drawing of the lot upon which his house was built, he saw that his lot was 75.38 feet wide. He said that the lot was about 75 feet wide.

1. Does 75.00 have the same value as 75? Does 76.00 have the same value as 76?
2. On the drawing below, which number is halfway between 75.00 and 76.00?

3. Mr. Jones rounded 75.38 to the nearest whole number. Is 75.38 nearer to 75 than to 76? How can you tell?
4. Use the diagram above to help you round each of the following to the nearest whole number.

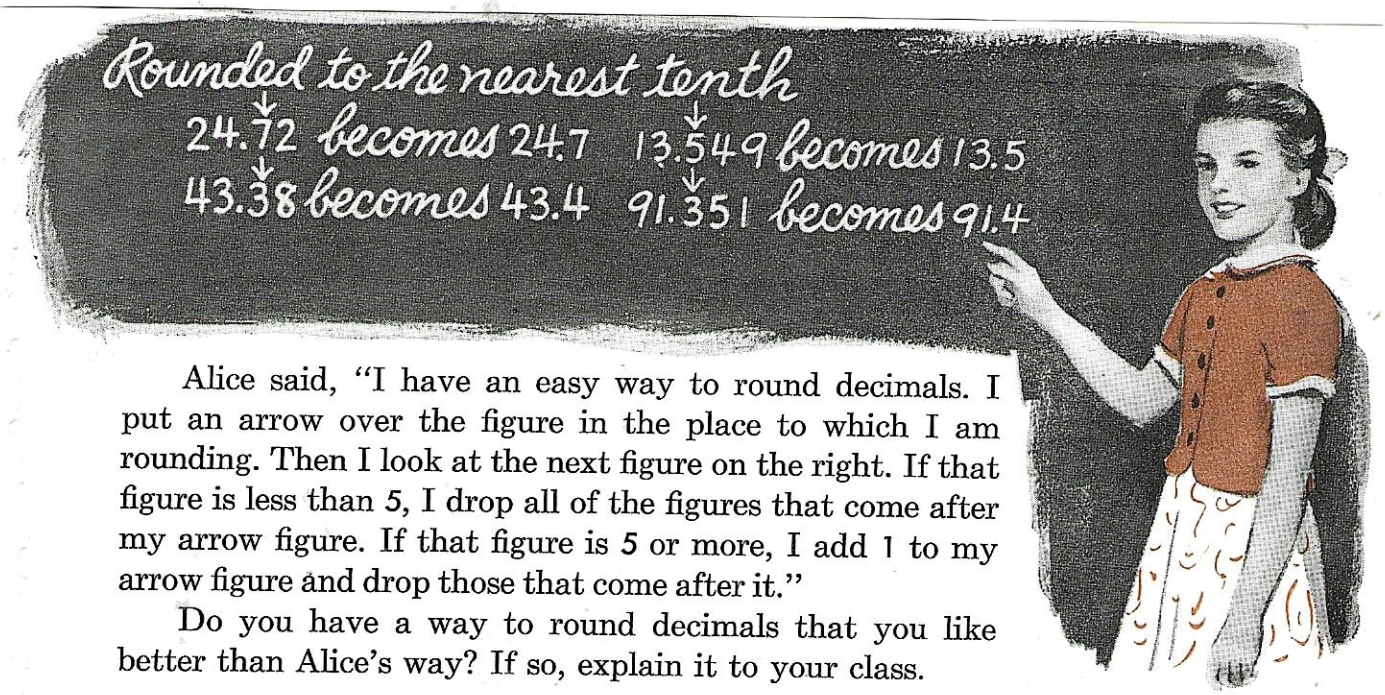
   75.60 75.65 75.21 75.90 75.13 75.49 75.51

5. Does 75.30 have the same value as 75.3? Does 75.40 have the same value as 75.4?
6. On the drawing below, which number is halfway between 75.30 and 75.40?

7. If Mr. Jones rounded 75.38 to the nearest tenth, he would say that his lot was 75.4 feet wide. Why?
8. Round each of the following to the nearest tenth. Make a drawing similar to the one in row 6 if you need help.

   75.32 75.62 75.48 75.33 81.58 12.24 24.09

9. What rule do you use for rounding whole numbers to the nearest ten? To the nearest hundred? To any place? Do you think you could use a similar rule with decimals?



Alice said, "I have an easy way to round decimals. I put an arrow over the figure in the place to which I am rounding. Then I look at the next figure on the right. If that figure is less than 5, I drop all of the figures that come after my arrow figure. If that figure is 5 or more, I add 1 to my arrow figure and drop those that come after it."

Do you have a way to round decimals that you like better than Alice's way? If so, explain it to your class.

Round each of the following to the nearest whole number.

10. 7.2 8.7 6.5 18.21 34.67 25.97 47.534

Round each of the following to the nearest tenth.

11. 5.37 8.34 15.35 .84 .750 92.076 .971

Round each of the following to the nearest hundredth.

12. 4.628 2.473 7.413 .565 18.845 .387 6.032

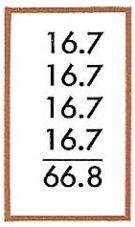
Estimating Products and Quotients

By rounding each decimal to the nearest whole number, tell which number after each problem below is nearest the correct answer.

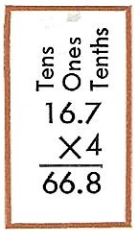
1. If the average cotton crop is 307.5 pounds per acre, a farmer who has planted 100 acres in cotton should expect to get a crop of about _?_ pounds. 300 300,000 30,000
2. If cotton is selling at 35.2¢ per pound, you should get about _?_ for 100 pounds. $.35 $35.00 $35,000
3. A man who picks 136.2 pounds of cotton in 3 hours is picking at the rate of about _?_ pounds an hour. $4\frac{1}{2}$ 45 450

Multiplying a Decimal or a Mixed Decimal by a Whole Number

1. Joe read that the average speed of a freight train is 16.7 miles per hour. He wondered how far the freight train could travel in 4 hours.

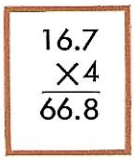


Joe found his answer by adding. When he added four .7's the sum was 28 tenths, or 2.8. He wrote 8 in tenths place. He kept the 2 to add with the other whole numbers. The answer was 66.8 miles. How many decimal places are there in Joe's answer?



Bill said, "I would multiply 4 × 16.7 to get the answer. I know that 4 × 16.7 is a little less than 4 × 17, or 68."

Bill multiplied 4 × 7 tenths = 28 tenths. He knew that 28 tenths equals 2 ones and 8 tenths. He wrote the 8 in tenths place and kept the 2 to add with the product of the ones. Then he multiplied the ones and tens remembering the numbers he carried. He put a decimal point in front of the 8 to show that this 8 means 8 tenths. The answer was 66.8 miles. He knew that his answer was sensible because it was a little less than 68. How many decimal places are there in the number Bill multiplied? How many decimal places in the product?



Jim said, "Why can't you multiply just as you multiply whole numbers; then put in a decimal point to show tenths in your answer?" Jim did his work as shown at the left. In his example, what number is the multiplicand? The multiplier? The product? What did Jim do to find his answer? How many decimal places are there in the answer?

2. Study the examples below. Think why there is one decimal place in each answer.

2 tenths	.2	4 tenths	.4	2.5
×2	×2	×6	×6	×7
4 tenths	.4	24 tenths	2.4	17.5
		or 2 and 4 tenths		

Find the following products by multiplying mentally. Write only the answers. Watch the decimal point.

3.	$2 \times .2$	$5 \times .2$	$11 \times .2$	$12 \times .2$
4.	$3 \times .3$	$3 \times .4$	$3 \times .5$	$5 \times .5$
5.	2×1.1	3×2.1	4×3.2	5×3.3

Copy and multiply.

6.	3.8	4.5	18.9	24.1	29.9
	4	9	2	5	9

7. In a recent month 3.68 inches of rain fell in New York. At this rate how much would fall in 3 months?

$$\begin{array}{r} 3.68 \\ \times 3 \\ \hline 11.04 \end{array}$$

To find his answer Bob multiplied 3×3.68. He thought: "3.68 is a little less than 4. $3 \times 4 = 12$, so my answer will be a little less than 12." Bob multiplied 3×3.68 as if the multiplicand were a whole number. Where should he put the decimal point so that his answer will be a little less than 12?

If you multiply hundredths by a whole number should you have hundredths in your answer?

Bob found that 11.04 inches of rain would fall. His answer was sensible because 11.04 is a little less than 12.

How many decimal places did Bob have in the multiplicand? How many decimal places in the product? What rule could Bob follow to find his answer?

8. Study the examples below. Would you say that multiplying numbers with decimal hundredths is like multiplying dollars and cents, except for the dollar sign? Copy the examples without the answers. Close your book and do them. Compare your answers with those in the book.

5.32	3.26	8.92	4.73	6.34
4	6	7	8	5
21.28	19.56	62.44	37.84	31.70

Now turn the page.

Copy and multiply.

9. $\begin{array}{r} 1.32 \\ \times\ 15 \\ \hline \end{array}$ $\begin{array}{r} 2.45 \\ \times\ 26 \\ \hline \end{array}$ $\begin{array}{r} 3.07 \\ \times\ 63 \\ \hline \end{array}$ $\begin{array}{r} 8.40 \\ \times\ 37 \\ \hline \end{array}$ $\begin{array}{r} 8.65 \\ \times\ 84 \\ \hline \end{array}$

10. $\begin{array}{r} .25 \\ \times\ 3 \\ \hline \end{array}$ $\begin{array}{r} .75 \\ \times\ 5 \\ \hline \end{array}$ $\begin{array}{r} .60 \\ \times\ 7 \\ \hline \end{array}$ $\begin{array}{r} .06 \\ \times\ 8 \\ \hline \end{array}$ $\begin{array}{r} .09 \\ \times\ 7 \\ \hline \end{array}$

11. $\begin{array}{r} .35 \\ \times\ 12 \\ \hline \end{array}$ $\begin{array}{r} .63 \\ \times\ 15 \\ \hline \end{array}$ $\begin{array}{r} .07 \\ \times\ 32 \\ \hline \end{array}$ $\begin{array}{r} .50 \\ \times\ 43 \\ \hline \end{array}$ $\begin{array}{r} .08 \\ \times\ 65 \\ \hline \end{array}$

12. In a recent year an automobile speed driver made an average of 125.135 miles an hour in a trial run at the Indianapolis Speedway. At this rate how many miles could he drive in 4 hours?

$$\begin{array}{r} 125.135 \\ \times\ 4 \\ \hline 500.540 \end{array}$$

To find the answer, Eddie thought: "125.135 is just a little more than 125, so my answer will be a little more than $4 \times 125 = 500$." Study the example at the left. Are there 3 decimal places in the multiplicand? Are there 3 decimal places in the product? Is the answer a little more than 500?

The driver could drive 500.540 miles in 4 hours.

What rule could Eddie have used to get his answer?

13. Study the examples below. Are there three decimal places in each multiplicand? Are there three decimal places in each product? Copy the examples without the answers. Close your book and work them. Compare your answers with those in the book.

$$\begin{array}{r} 3.122 \\ \times\ 8 \\ \hline 24.976 \end{array} \qquad \begin{array}{r} .805 \\ \times\ 6 \\ \hline 4.830 \end{array} \qquad \begin{array}{r} .041 \\ \times\ 5 \\ \hline .205 \end{array} \qquad \begin{array}{r} 5.213 \\ \times\ 31 \\ \hline 5\,213 \\ 156\,39 \\ \hline 161.603 \end{array}$$

Find the products.

14. $\begin{array}{r} .912 \\ \times\ 4 \\ \hline \end{array}$ $\begin{array}{r} .108 \\ \times\ 52 \\ \hline \end{array}$ $\begin{array}{r} .036 \\ \times\ 8 \\ \hline \end{array}$ $\begin{array}{r} .037 \\ \times\ 8 \\ \hline \end{array}$ $\begin{array}{r} .855 \\ \times\ 6 \\ \hline \end{array}$ $\begin{array}{r} 2.453 \\ \times\ 9 \\ \hline \end{array}$

15.

5.913	3.425	8.413	5.236	16.231	21.561
5	15	78	32	12	31

Eddie said, "I can make a rule for multiplying any decimal or mixed decimal by a whole number."

This is his rule.

> To multiply any decimal or mixed decimal by a whole number, multiply as you would if both numbers were whole numbers. Then point off as many decimal places in the product as there are decimal places in the multiplicand.

Tell how many decimal places there will be in each product. Then multiply and check.

16. 7×1.96 8×3.122 31×5.213 $71 \times .21$
17. 6×22.3 21×19.2 5×9.14 10×9.87
18. 12×4.12 24×2.03 6×80.5 $80 \times .307$

Solve the following problems.

19. Jim's sister travels 6.5 miles each way to work. How many miles does she travel in a week of 5 work days?
20. In 1921 the average speed of a freight train was 11.5 miles per hour. At that rate how far could a freight train go in 12 hours?
21. Today the average speed of a freight train is 16.7 miles per hour. At that rate how far can a freight train travel in 12 hours?
22. The mail section of a combination baggage and mail car is 39.75 feet by 9 feet. What is the area of the mail section?
23. Train fare from John's home town to Washington, D. C. is \$30.74 per round trip. How much will it cost John's father during the year for train fare if he makes 6 business trips to Washington?

Multiplying a Whole Number by a Decimal or a Mixed Decimal

1. Ann read that a soil conservation expert said it took nature 600 years to make an inch of topsoil in a part of the United States. How many years would it take to make 4.5 inches of topsoil?

```
 60 0
  4.5
------
300 0
2400
------
2700.0
```

To find the answer, Ann multiplied 600 by 4.5. She thought: "4.5 is more than 4 but less than 5, so my answer will be more than $4 \times 600 = 2400$ and less than $5 \times 600 = 3000$." Ann multiplied as she would multiply whole numbers. Where will Ann put the decimal point so that her answer will be between 2400 and 3000? It will take nature 2700 years to make 4.5 inches of topsoil.

How many decimal places did Ann have in her multiplier? In the product?

You know that interchanging the multiplier and the multiplicand does not change the product when the numbers you are multiplying are whole numbers or fractions. Would you expect the same thing to be true if one of the numbers were a decimal?

Change the order of the numbers in Ann's example and multiply again. Did you get 2700.0 for an answer?

Make up a rule for multiplying a whole number by a decimal or a mixed decimal. Use it to do the following examples. Copy and multiply. Watch the decimal points.

2.	3	4	65	130	345
	.3	.8	.7	.6	.4
3.	2 3	3 7	4 6	18 2	56 7
	1.5	2.8	3.7	5.7	2.9
4.	25	36	92	130	369
	.05	.08	.26	.57	.09

5.	2 36	3 74	5 40	7 08	8 00
	1.02	1.26	3.63	2.15	1.07
6.	45	236	576	1347	3 462
	.007	.106	.231	.204	1.345

7. Check the examples in row 5 by interchanging the multiplier and the multiplicand and multiplying again.

8. Mary has a scarf that is 36 inches long. Ann has a scarf that is 1.5 times as long as Mary's. How long is Ann's scarf?

Just for Fun

The problems below, with their answers, appeared in an arithmetic book that is about 150 years old. See if you can solve these problems, just for fun.

1. There were forty-one men concerned in the payment of a sum of money, and each paid 3 dollars and 9 mills; how much was paid in all?

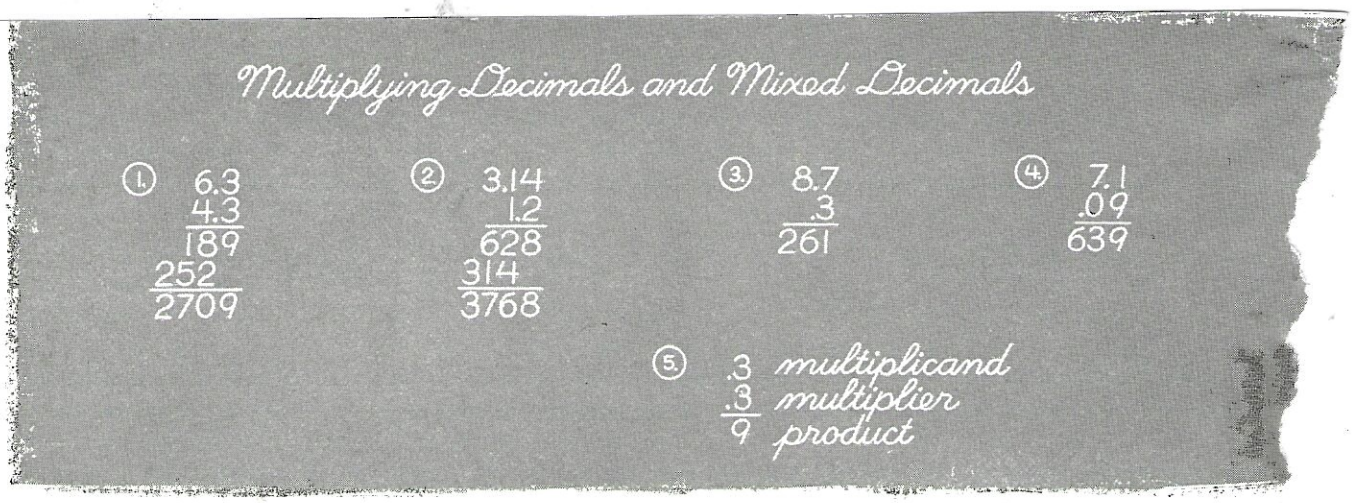
   Ans. $123 36 cts. 9 mills.

2. The number of inhabitants in the United States is five millions; now suppose each should pay the trifling sum of 5 cents a year, for the term of 12 years, towards a continental tax; how many dollars would be raised thereby?

   Ans. three million Dollars.

3. I lent Peter Trusty 1000 dols. and afterwards lent him 26 dols. 45 cts. more. He has paid me at one time 361 dols. 40 cts. and at another time 416 dols. 09 cts. besides a note which he gave me upon James Paywell, for 143 dols. 90 cts.: how stands the balance between us?

   Ans. The balance is $105 06 cts. due to me.


Multiplying Decimals and Mixed Decimals
① 6.3 × 4.3 = 189 + 252 = 2709
② 3.14 × 1.2 = 628 + 314 = 3768
③ 8.7 × .3 = 261
④ 7.1 × .09 = 639
⑤ .3 multiplicand
.3 multiplier
9 product

On a piece of paper, copy the examples that are written on the blackboard in the picture. The examples are not finished. Miss White asked her class to see if they could develop a rule for placing the decimal point in the answers when both numbers being multiplied contain decimals.

Alice said, "I can tell where to put the decimal point in the first example by estimating. 4.3×6.3 should be a little more than $4 \times 6 = 24$. We should put the decimal point after the 7, because 27.09 is a little more than 24."

1. Place the decimal point correctly in the first example on your paper.
2. Use Alice's way to tell where to put the decimal point in the second example.

Now look at the third example. John said, "I think it is hard to estimate answers with numbers less than 1. I'll try writing the numbers in this example with common fractions in place of the decimals."

3. Write .3 as a common fraction. Write 8.7 as a mixed number. Multiply $\frac{3}{10} \times 8\frac{7}{10}$. Change $\frac{261}{100}$ to a mixed number. In the third example on your paper, place the decimal point so that the answer will mean the same as $2\frac{61}{100}$.
4. Use John's idea to tell where to place the decimal point in the fourth and fifth examples on your paper. Why do you have to write a zero in front of the 9 in the fifth example?

5. Study the five completed examples on your paper. For each example, answer these questions: (a) How many decimal places are there in the multiplicand? In the multiplier? In the product? (b) Are there as many decimal places in the product as there are decimal places in the multiplier and the multiplicand together?

6. Make a rule for multiplying two numbers that contain decimals. Compare your rule with the rule below.

> Multiply decimals and mixed decimals just as you multiply whole numbers. Then point off as many decimal places in the product as there are decimal places in the multiplier and the multiplicand together. Write zeros to the left of the product when they are needed to make the required number of decimal places.

Copy the following examples. Place the decimal point correctly in the products.

7.

3.4	.5	.8	.3	.35	.01
1.2	.7	.5	.3	.7	.5
408	35	40	9	245	5

Copy and multiply. Take care with the decimal points.

8.

.8	.2	.44	.05	.01
.7	.4	.6	.3	.9

9.

2.6	3.7	.59	62.8	8.3
.4	.5	.6	4.7	.09

10.

1.34	7.5	7.6	3.27	1.02
.9	.04	5.4	2.3	8.3

11.

54.7	17.3	3.45	27.1	7.08
.12	4.7	2.7	3.24	.7

12. Can you use the rule on this page if one of the numbers you are multiplying is a whole number?

Practice to Remember

MAINTENANCE TEST ON MEANINGS AND SKILLS (CHAPTERS 1–8)

1. Write in figures: One hundred sixty-seven million, three hundred four thousand, two hundred fifty.
2. Use the following example to show that changing the order of the addends does not change the sum: $465 + 728 + 391$
3. Round the following numbers to the nearest ten thousand:

   34,652 58,269 86,200 64,915
4. Change the following to improper fractions: $3\frac{2}{3}$ $8\frac{4}{5}$ $15\frac{3}{8}$
5. Change the following to whole numbers or mixed numbers:

   $\frac{15}{5}$ $\frac{16}{3}$ $\frac{26}{8}$ $\frac{36}{12}$ $\frac{15}{9}$ $\frac{55}{10}$ $\frac{18}{18}$ $\frac{68}{16}$
6. Find the area of a rectangle 26 ft. by 34 ft.
7. Find the perimeter of a square 18 in. on a side.
8. Add 3 hr. 25 min. and 6 hr. 50 min. Divide this sum by 5.
9. Subtract 5 yd. 2 ft. from 8 yd. 1 ft. Multiply the difference by 8.
10. 16 pk. = _?_ bu. 5 da. = _?_ hr. 72 sq. ft. = _?_ sq. yd.
11. Subtract: 9872 from 14681 $867.75 from $1556.00

Copy and do as the signs tell you. Be careful.

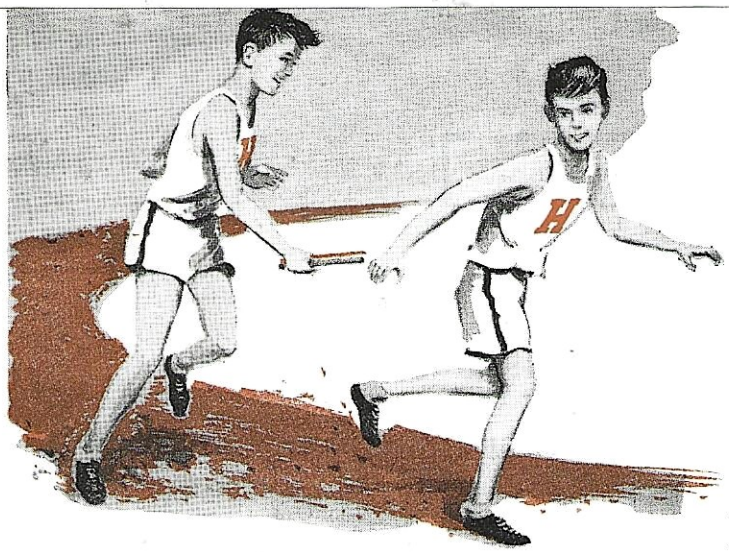
12. $\begin{array}{r} 457 \\ +62 \\ \hline \end{array}$ $\begin{array}{r} 3462 \\ -86 \\ \hline \end{array}$ $\begin{array}{r} 536 \\ \times 700 \\ \hline \end{array}$ $\begin{array}{r} 570 \\ \times 304 \\ \hline \end{array}$ $\begin{array}{r} 307 \\ \times 503 \\ \hline \end{array}$
13. $63\overline{)1584}$ $38\overline{)3524}$ $84\overline{)3672}$ $64\overline{)38642}$ $160\overline{)552}$
14. $19\frac{5}{6} + 8\frac{2}{3}$ $12\frac{1}{2} + 6\frac{3}{8}$ $8\frac{1}{4} + 5\frac{2}{3}$ $6\frac{1}{3} + 8\frac{3}{5}$
15. $8\frac{7}{8} - 4\frac{1}{4}$ $17\frac{1}{6} - 4\frac{5}{6}$ $23\frac{3}{4} - 9\frac{2}{3}$ $34\frac{3}{10} - 7\frac{3}{5}$
16. $\frac{5}{7} \times \frac{14}{15}$ $4\frac{2}{3} \times \frac{3}{7}$ $5\frac{2}{5} \times 25$ $5\frac{2}{3} \times 8\frac{1}{2}$
17. $\frac{5}{6} \div 5$ $15 \div 7\frac{1}{2}$ $6\frac{1}{4} \div \frac{5}{9}$ $5\frac{1}{3} \div 2\frac{2}{9}$

Practice to Remember

MAINTENANCE TEST ON PROBLEM SOLVING (CHAPTERS 1–8)

Read each problem carefully. Decide when to add, subtract, multiply, or divide. Then find the answer.

1. Tom had 309 foreign stamps and 232 United States stamps in his collection. His uncle gave him 175 foreign stamps and 121 United States stamps. How many stamps in all are in his collection now?
2. One day in Flagstaff the temperature reached 112.8 degrees. After sundown the temperature fell to 96.2 degrees. How many degrees did the temperature fall?
3. Betty's club made 480 pieces of fudge for the Girl Scout fair. They put the fudge into 24 boxes of the same size. How many pieces did they put in each box?
4. Joe watched television $1\frac{3}{4}$ hours on Saturday afternoon and $2\frac{1}{2}$ hours that evening. How long did he watch television that day?
5. Bill carried two packages to the post office. One weighed $8\frac{3}{4}$ pounds, the other $6\frac{1}{8}$ pounds. How much less did the lighter package weigh?
6. Mary's mother bought three chickens for the picnic. One weighed 2 pounds 3 ounces, another 3 pounds 6 ounces, and the third 2 pounds 7 ounces. What was their total weight?
7. A one-quart measure holds 1 pound 2 ounces of corn meal. How many pounds and ounces of corn meal are there in 8 quarts?
8. How much is $\frac{2}{3}$ of $7\frac{1}{2}$ cups of flour?
9. Turkeys are on sale at 48 cents a pound. How much will a turkey weighing $12\frac{3}{4}$ pounds cost?
10. Judith has 2 pounds of mints to put in bags of $\frac{1}{8}$ pound each. How many bags of mints can she make?
11. Tom's father took him for a ride in his motorboat. They traveled $2\frac{1}{2}$ miles in $8\frac{3}{4}$ minutes. How many miles did they travel in one minute?
12. Tom caught a fish weighing $3\frac{3}{4}$ pounds when ready to cook. How many portions of $\frac{3}{4}$ pound each can be served from this fish?



Dividing a Decimal by a Whole Number

1. John and Bill read that a boys' team ran 4 laps around the track in 64.8 seconds. They wondered what the average time for a lap was.

To find the answer, they knew that they should divide 64.8 by 4.

Bill said, "Why not divide as you divide whole numbers and then put the decimal point in the quotient to make the answer sensible? The answer would be a little more than 16 because $4\overline{)64}$ is 16."

Where would Bill put the decimal point in his quotient? The average time for a lap was 16.2 seconds.

```
  16.2
4)64.8
  4
  24
  24
   8
   8
```

2. Kay rode her bicycle 18.36 miles in 3 hours. On an average, how many miles did she ride in an hour?

Bill thought: "The answer will be a little more than 6 because $3\overline{)18}$ is 6."

Where will Bill put the decimal point to make the answer sensible?

```
  6.12
3)18.36
  18
   3
   3
   6
   6
```

John said, "Look at those examples. When you divide tens, you write the quotient figure in tens place. When you divide ones, you write the quotient figure in ones place. When you divide tenths, you write the quotient figure in tenths place, and so on. The decimal point in the quotient goes right above the decimal point in the dividend."

To divide a decimal by a whole number, write a decimal point in the quotient directly above the decimal point in the dividend. Divide just as if the dividend were a whole number.

Copy the following examples just as they are in the book. Then place the decimal point correctly in each quotient. Think how each quotient figure is found. Think why it is necessary to write a zero in some of the quotients.

3.

4	4	32	06	08
2)$.8	4)1.6	2)$.64	4)$.24	7)$.56

4.

3 6	15	1 05	2 08	02
6)21.6	2)$.30	5)5.25	8)16.64	91)1.82
18	2	5	16	1 82
3 6	10	25	64	
3 6	10	25	64	

Copy and divide. Check by multiplying.

5.	2)$.4	3)$.9	4)$.8	2)$.6	7)$.7
6.	2)1.2	3)1.2	5)1.5	6)1.8	8)2.4
7.	5)2.0	7)3.5	9)5.4	8)7.2	6)4.8
8.	3)15.6	6)26.4	8)42.4	5)37.5	9)83.7
9.	4)$.48	6)$.36	3)$.69	5)$.45	8)$.64
10.	2)4.54	3)1.59	5)3.65	6)4.38	7)15.68
11.	4)$.08	9)$.27	6)3.84	2)6.08	5)30.45
12.	7)$.49	4)1.56	5)15.25	3)9.27	8)$.40
13.	5)16.5	8)22.4	6)21.6	7)38.5	4)23.6
14.	12)85.2	23)186.3	34)176.8	14)47.6	71)248.5
15.	5)17.75	3)24.96	7)50.61	8)49.04	4)49.40
16.	7)69.65	11)12.65	24)19.44	23)11.73	45)19.35
17.	7)37.457	6)58.572	8)72.976	12)25.608	15)9.765

Changing Fractions to Decimals

1. Fill in the missing numerators.

$\frac{1}{2} = \frac{?}{10}$ $\frac{4}{5} = \frac{?}{10}$ $\frac{1}{4} = \frac{?}{100}$ $\frac{2}{25} = \frac{?}{100}$ $\frac{1}{8} = \frac{?}{1000}$

2. Write the following fractions as decimals: $\frac{3}{10}$ $\frac{6}{10}$ $\frac{8}{10}$

$\frac{25}{100}$ $\frac{75}{100}$ $\frac{1}{100}$ $\frac{125}{1000}$ $\frac{875}{1000}$ $\frac{38}{1000}$ $\frac{9}{1000}$

In a large city, the weather bureau reported that on a day in May $\frac{1}{4}$ inch of rain fell. John said that $\frac{1}{4}$ inch is equal to .25 inch.

He said, "I can change fourths into hundredths by multiplying both numerator and denominator by 25. $\frac{1}{4} = \frac{25 \times 1}{25 \times 4} = \frac{25}{100}$. I know that the common fraction $\frac{25}{100}$ may be written as the decimal fraction .25."

```
   .25
4)1.00
   8
   20
   20
```

3. Miss White told the pupils that they could change the fraction $\frac{1}{4}$ to a decimal in another way. She said, "We know that the common fraction $\frac{1}{4}$ may mean $1 \div 4$. To divide 1 by 4, we write a decimal point and zeros to the right of the 1. Does 1.00 have the same value as 1? Then we divide."

Study the top example at the left. Why did Miss White use two zeros?

```
   .8
5)4.0
  4 0
```

4. Change $\frac{4}{5}$ to a decimal. $\frac{4}{5} = 4 \div 5$. Write a decimal point and a zero after the 4 and then divide. Why is one zero enough in this example?

```
   .625
8)5.000
  4 8
   20
   16
   40
   40
```

5. To change $\frac{5}{8}$ to a decimal, think of $\frac{5}{8}$ as $5 \div 8$. Write a decimal point and zeros to the right of the 5 and divide. How many zeros are needed in this example?

To change a fraction to a decimal, write the numerator with a decimal point and one or more zeros after it. Then divide by the denominator.

Change the following fractions to decimals. Write zeros to the right of the decimal point until the division comes out even.

6.	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{3}{10}$	$\frac{3}{8}$	$\frac{7}{20}$	$\frac{3}{4}$
7.	$\frac{1}{5}$	$\frac{17}{20}$	$\frac{7}{40}$	$\frac{3}{20}$	$\frac{3}{5}$	$\frac{9}{20}$	$\frac{1}{20}$
8.	$\frac{11}{50}$	$\frac{7}{8}$	$\frac{9}{10}$	$\frac{13}{20}$	$\frac{2}{5}$	$\frac{7}{25}$	$\frac{3}{16}$

When some fractions are changed to decimals, there is always a remainder, no matter to how many places the division is carried. The fraction $\frac{1}{3}$ shows this.

$$3\overline{)1.0}\quad .3\tfrac{1}{3} \qquad 3\overline{)1.00}\quad .33\tfrac{1}{3} \qquad 3\overline{)1.000}\quad .333\tfrac{1}{3} \qquad 3\overline{)1.0000}\quad .3333\tfrac{1}{3}$$

Such decimals are called *repeating decimals* because the figures repeat. The fraction $\frac{1}{3}$ changes to a decimal that repeats after the first decimal place. Some fractions with large denominators will change to a decimal that runs to many decimal places before the figures begin to repeat.

Change the following fractions to decimals. Express your answers in hundredths with a fraction, such as $\frac{1}{3}$ equals $.33\frac{1}{3}$.

9.	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{9}$	$\frac{2}{3}$	$\frac{4}{9}$	$\frac{4}{15}$	$\frac{5}{9}$
10.	$\frac{3}{7}$	$\frac{2}{7}$	$\frac{4}{7}$	$\frac{5}{12}$	$\frac{5}{6}$	$\frac{7}{9}$	$\frac{3}{14}$
11.	$\frac{7}{15}$	$\frac{2}{15}$	$\frac{1}{11}$	$\frac{5}{7}$	$\frac{6}{7}$	$\frac{8}{9}$	$\frac{13}{15}$

✪ Some fractions occur often in daily work. If you know the decimals which are equal, or equivalent, to these fractions, you will be able to work faster. Learn the following fractions and their decimal equivalents.

$\frac{1}{2} = .5$ or $.50$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{6} = .16\frac{2}{3}$
$\frac{1}{4} = .25$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{1}{5} = .2$ or $.20$
$\frac{3}{4} = .75$	$\frac{1}{8} = .125$ or $.12\frac{1}{2}$	$\frac{7}{8} = .875$ or $.87\frac{1}{2}$

A Mixed Decimal in the Quotient

$$\begin{array}{r} 4\frac{1}{4} \\ 4\overline{)17} \\ 16 \\ \hline 1 \end{array}$$

Mr. Scott works on a machine that will adjust to cutting lengths in whole inches and decimal fractions of an inch. He has a metal rod 17 inches long that he wants to cut into 4 equal pieces. To what number should he set his machine?

He can find out by dividing as shown at the left. What kind of number is the quotient?

Mr. Scott needs his answer in decimal form. How can he change $\frac{1}{4}$ to a decimal? What is the answer in decimal form?

$$\begin{array}{r} 4.25 \\ 4\overline{)17.00} \\ 16 \\ \hline 10 \\ 8 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

He could do the same thing in one example. He could place a decimal point and zeros after the 17 in the dividend and continue to divide. Study the example at the left. Where is the decimal point placed in the quotient? Find the parts of the example that show $1.00 \div 4$. Is 4.25 equal to $4\frac{1}{4}$?

Mr. Scott should set his machine at 4.25 inches.

The fractional part of a quotient may be found in decimal form by writing a decimal point and zeros after the whole number in the dividend, writing a decimal point in the quotient, and continuing the division.

Write each answer in decimal form.

1. During a summer storm 3 inches of rain fell in 2 hours. What was the average rainfall per hour?
2. Mr. Ferris harvested 780 bushels of wheat from his field of 24 acres. On an average, how many bushels of wheat did he get from each acre?
3. Mrs. Richards used 9 pounds of butter in 4 weeks. On an average, how much butter did she use in a week?

4. $5\overline{)9}$ $8\overline{)37}$ $20\overline{)87}$ $16\overline{)51}$ $25\overline{)82}$
5. $8\overline{)55}$ $12\overline{)39}$ $16\overline{)232}$ $32\overline{)368}$ $20\overline{)149}$

Rounding Decimals in Quotients

A decimal in a quotient may be rounded to any required number of decimal places. In real-life problems, we are able to judge how many decimal places we need to suit our purpose.

Think about the following cases: (a) Your father is eager to find out how many miles he gets in his car on a gallon of gasoline. Can he measure the mileage closer than to the nearest tenth of a mile? Can he measure the gasoline used closer than to the nearest tenth of a gallon? Would a figure to the nearest tenth, such as 18.9 miles per gallon, be accurate enough for his purpose? (b) Cars made by different manufacturers are being tested for gasoline economy. They will all use the same kind of gasoline. The cars are run over a test route that has been measured carefully. The gasoline is measured in a specially constructed flask so that the testers can tell within a few drops the amount of gasoline used. Would a figure to the nearest thousandth, such as 21.307 miles per gallon, be reasonable in this case? Might it be necessary to have a figure to three decimal places in order to compare the performance of two cars?

1. What rule do you use for rounding decimals? In rounding to the nearest tenth, you have to see what number is in tenths place and what number is in _?_ place. In rounding to the nearest hundredth, you have to see what number is in hundredths place and what number is in _?_ place. Do you always need one place to the right of the place to which you are rounding?

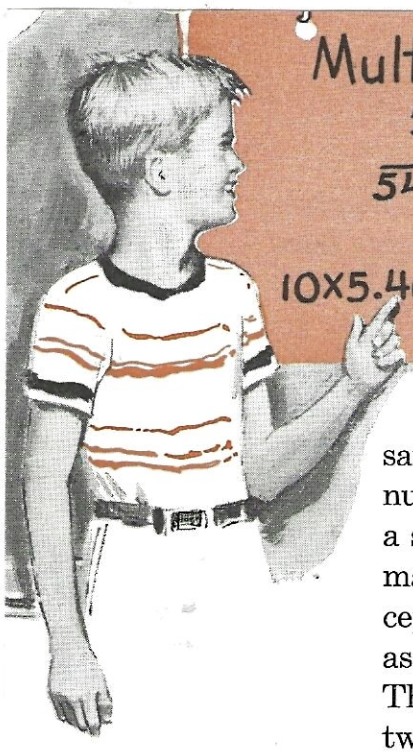
> To round a decimal in a quotient, carry the division to one more place than will be kept in the quotient. Then round the quotient as you would round any decimal.

2. Copy. Divide. Round to the nearest tenth.

$3\overline{)4}$ $9\overline{)10}$ $9\overline{)60}$ $8\overline{)11}$ $21\overline{)34}$ $46\overline{)29.1}$ $52\overline{)162.83}$

3. Copy. Divide. Round to the nearest hundredth.

$3\overline{)2}$ $9\overline{)17}$ $16\overline{)62}$ $45\overline{)32}$ $34\overline{)73.7}$ $37\overline{)162.83}$



Multiplying Decimals by 10, 100, 1000

$$\begin{array}{r} 5.46 \\ \times\ 10 \\ \hline 54.60 \end{array} \qquad \begin{array}{r} 5.46 \\ \times\ 100 \\ \hline 546.00 \end{array} \qquad \begin{array}{r} 5.46 \\ \times\ 1000 \\ \hline 5460.00 \end{array}$$

A short cut to use

$10 \times 5.46 = 54.6$ $100 \times 5.46 = 546.$ $1000 \times 5.46 = 5460.$

Tom brought to class a chart he had made at home. He said, "I knew that we have a short way to multiply whole numbers by 10, 100, or 1000. I wondered if there might be a short way to multiply decimals by these numbers. I tried many examples before I made my chart. I noticed that, except for some extra zeros, the product had the same figures as the multiplier. The decimal point was in a different place. The decimal point was moved to the right one place for 10, two places for 100, and three places for 1000."

Study Tom's chart. Do you understand what he said? Use the numbers on the chart to explain the following rule.

Multiplying a decimal or a mixed decimal by 10, 100, or 1000 moves the decimal point in the numeral as many places to the right as there are zeros in the multiplier. Missing places are filled in with zeros.

Multiply. Write the products only.

1. 10×38.6 100×3.7 $1000 \times .34$
2. 10×4.271 $100 \times .85$ 1000×6.8
3. Multiply each of the following numbers by 10.

   4.5 3.62 4.07 .65 2.134 .09 .642
4. Multiply each of the following numbers by 100.

   4.5 1.27 2.06 .74 3.162 .056 31.4
5. Multiply each of the following numbers by 1000.

   8.1 .462 1.27 2.06 .7 86.5 .076

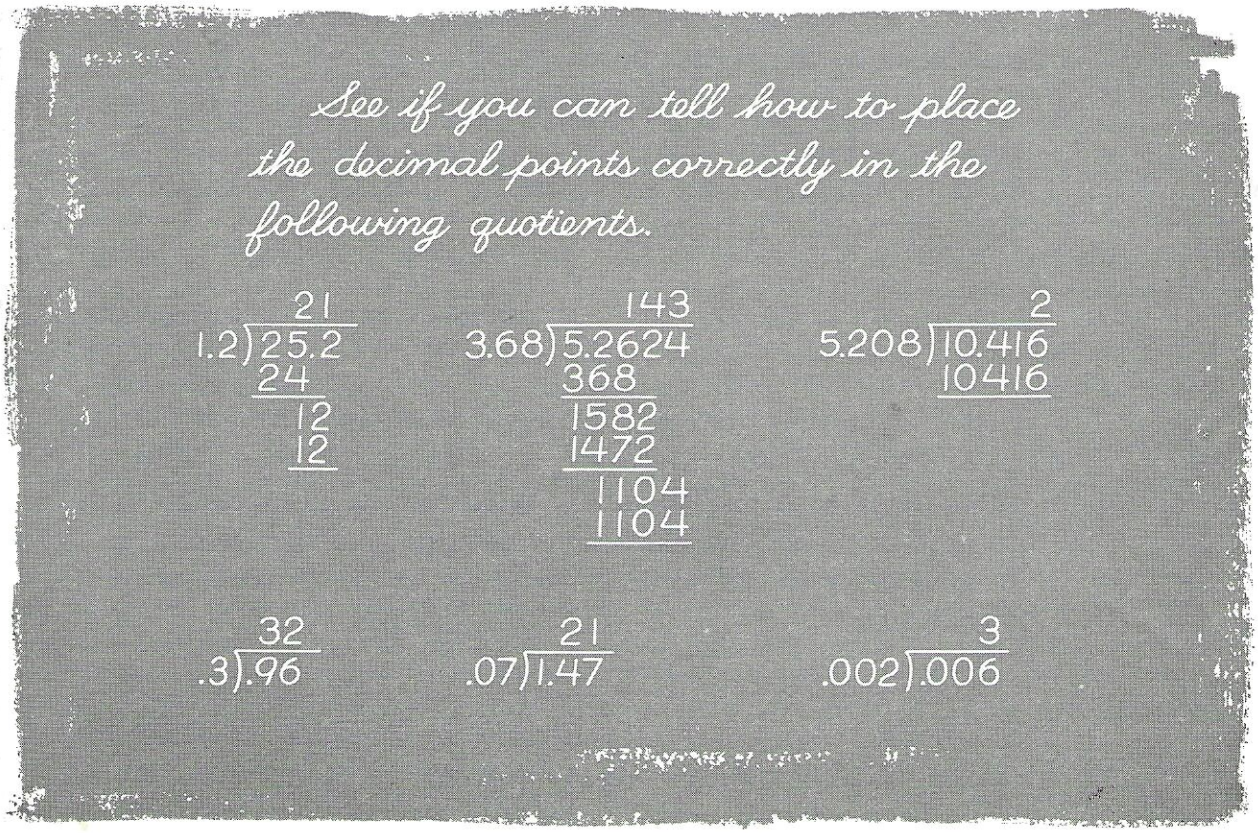
Dividing by a Decimal

Copy the examples shown in the picture below. How would you decide where to place the decimal point in each quotient?

Joe said, "We could estimate each quotient, then place the decimal point so that the answers would be near our estimates."

Kay said, "We could write each decimal as a common fraction. Then divide, and change the answers back to decimal form."

1. Use Joe's idea to place the decimal points correctly in the examples in the top row.
2. Use Kay's idea to place the decimal points correctly in the examples in the bottom row.
3. Check each quotient by multiplying the quotient and the divisor. How can you tell whether your quotient was correct?



Now see what happens to the quotient in a division example if we multiply both the divisor and the dividend by the same number. Study the examples below. Discover what has been done to the divisor and the dividend of the first example in each row to produce the other examples in that row. Compare the quotients.

$3\overline{)9}$	$6\overline{)18}$	$21\overline{)63}$	$36\overline{)108}$
		63	108
$5\overline{)10}$	$15\overline{)30}$	$50\overline{)100}$	$250\overline{)500}$
	30	100	500

4. Write any division example, such as $18 \div 9$, as a fraction. Is the value of a fraction changed if both numerator and denominator are multiplied by the same number? Then is the value of a quotient changed if both the dividend and divisor are multiplied by the same number?
5. Compare each of the last three examples below with the first. How do you multiply mentally by 10, 100, or 1000? If both the dividend and divisor are multiplied by 10, 100, or 1000, does the quotient remain the same?

$2\overline{)6}$	$20\overline{)60}$	$200\overline{)600}$	$2000\overline{)6000}$
	60	600	6000

6. Explain what has been done to the first example in each pair of divisions below to produce the second example. Will the two quotients in each pair be equal? What kind of numbers are the divisors in the second example of each pair? Where would the decimal point be placed in the quotient of the second example of each pair?

$3.4\overline{).68} \rightarrow 34.\overline{)6.8}$ $\quad .02\overline{).14} \rightarrow 2.\overline{)14.}$ $\quad .123\overline{)2.36} \rightarrow 123.\overline{)2360.}$

This gives us an easy way to divide by a decimal because we can make the divisor into a whole number by performing a simple multiplication which we can do mentally.

Copy each example below. Multiply the divisor and the dividend by 10. Rewrite and divide.

7. $.7\overline{)3.5}$ $.6\overline{).84}$ $1.5\overline{)7.5}$ $.5\overline{)3}$ $1.4\overline{).28}$

Copy each example below. Multiply the divisor and the dividend by 100. Rewrite and divide.

8. $.04\overline{).08}$ $.05\overline{).165}$ $2.31\overline{)4.62}$ $.08\overline{)1.6}$ $.21\overline{).231}$

Copy each example below. Multiply the divisor and the dividend by 1000. Rewrite and divide.

9. $.002\overline{).008}$ $.003\overline{).1269}$ $.036\overline{).756}$ $.025\overline{).05}$ $.241\overline{).5543}$

Instead of rewriting the example each time, we can save time by using a *caret* ($\wedge$) to show the new position of the decimal point. Use the example below to help you understand the following rule.

(1) $7.1\wedge\overline{)24.85}$ (2) $7.1\wedge\overline{)24.8\wedge5}$ (3) $7.1\wedge\overline{)24.8\wedge\dot{5}}$

To divide by a decimal: (1) Multiply the divisor by 10, 100, or 1000 to make the divisor a whole number. Show this by a caret. (2) Multiply the dividend by the same number. Show this by a caret. (3) Place the decimal point in the quotient above the caret in the dividend. (4) Divide as you divide by a whole number.

In rows 10 and 11, the first two steps are done for you. Think why the carets are placed correctly. Then copy, finish each example, and check.

10. $.4\wedge\overline{)1.6\wedge}$ $.4\wedge\overline{).3\wedge2}$ $2.3\wedge\overline{)1.6\wedge1}$ $1.2\wedge\overline{)14.4\wedge}$

11. $.05\wedge\overline{).75\wedge}$ $.36\wedge\overline{).75\wedge6}$ $.31\wedge\overline{)6.20\wedge}$ $.074\wedge\overline{)1.776\wedge}$

Copy and divide each example below. Use the rule. Always take great care with the decimal point.

12. $.3\overline{)1.5}$ $.5\overline{).65}$ $.21\overline{).441}$ $.007\overline{)1.4847}$

Practice in Division with Decimals

The examples and problems below will give you practice in dividing with decimals. If a division is not complete by the third decimal place, round the quotient to the nearest hundredth.

1.	$3\overline{).9}$	$4\overline{).8}$	$2\overline{)6.8}$	$9\overline{)18.9}$
2.	$4\overline{)9.24}$	$8\overline{)5.28}$	$9\overline{)6.57}$	$6\overline{)9.36}$
3.	$7\overline{)23.457}$	$6\overline{)2.124}$	$4\overline{)18.344}$	$5\overline{)37.215}$
4.	$8\overline{).976}$	$4\overline{).736}$	$7\overline{).735}$	$3\overline{).612}$
5.	$3\overline{).15}$	$8\overline{).48}$	$7\overline{).049}$	$4\overline{).072}$
6.	$8\overline{)12}$	$5\overline{)4}$	$16\overline{)1.2}$	$26\overline{)3.9}$
7.	$36\overline{)63}$	$16\overline{)24}$	$48\overline{)54}$	$25\overline{)37}$
8.	$.2\overline{)8}$	$.5\overline{)45}$	$.6\overline{)246}$	$.9\overline{)81}$
9.	$.04\overline{)32}$	$.12\overline{)84}$	$.26\overline{)156}$	$.54\overline{)342}$
10.	$.5\overline{)1.5}$	$.7\overline{)4.9}$	$2.3\overline{)6.9}$	$3.4\overline{)10.2}$
11.	$.15\overline{)2.25}$	$.23\overline{)12.88}$	$.34\overline{)8.16}$	$.06\overline{)13.2}$
12.	$.02\overline{).004}$	$.51\overline{)1.734}$	$.13\overline{)1.056}$	$.37\overline{).4452}$
13.	$3.2\overline{)14.72}$	$12.2\overline{)5.734}$	$8.21\overline{)73.89}$	$9.3\overline{)14.82}$
14.	$.07\overline{)1.41}$	$3.6\overline{)6.62}$	$.03\overline{)8.61}$	$.43\overline{)14.2}$

15. Walter made 23 hits out of 72 times at bat. Hits ÷ times at bat = batting average. What is Walter's batting average? Express your answer as a decimal rounded to the nearest thousandth.

16. A certain kind of paper is .025 inch thick. How many sheets of this paper packed tightly together would make a stack 3 inches high?

Dividing Decimals by 10, 100, and 1000

When you multiply numbers by 10, 100, and 1000, you can multiply mentally by moving the decimal point to the right. For example, $5.46 \times 10 = 54.6$; $5.46 \times 100 = 546$; $5.46 \times 1000 = 5460$.

Do you think there might be a similar way to divide by 10, 100, and 1000? See if you can get a hint from the examples below.

```
   3.54          .354          .0354
10)35.40      100)35.400    1000)35.4000
  30            30 0          30 00
   5 4           5 40          5 400
   5 0           5 00          5 000
    40            400           4000
    40            400           4000
```

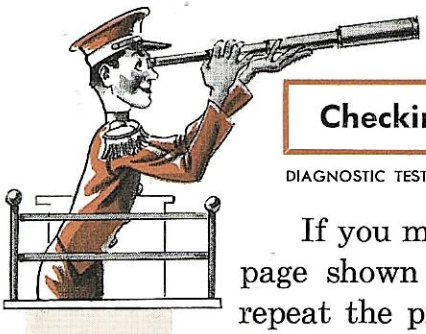
In each division example above, compare the dividend with the quotient. In each example, does the quotient have the same figures as the dividend, except for some zeros? How has the position of the decimal point been changed? Why is it necessary to write a zero between the decimal point and the 3 in the quotient of the third example? Do you see a short cut to use for divisions like these?

> Dividing a number by 10, 100, or 1000 moves the decimal point in the numeral as many places to the left as there are zeros in the divisor. Missing places are filled in with zeros.

1. If you want to write a whole number with a decimal point, where will you place the decimal point?

Divide. Write only the quotients.

2. $65.3 \div 10$	$1.54 \div 10$	$347 \div 10$
3. $34.7 \div 100$	$.27 \div 100$	$476 \div 100$
4. $2.9 \div 1000$	$52.2 \div 1000$	$72 \div 1000$
5. $3.76 \div 10$	$4.52 \div 100$	$86.4 \div 1000$
6. $7 \div 10$	$8 \div 100$	$9 \div 1000$



Checking Up on Multiplying and Dividing Decimals

DIAGNOSTIC TEST ON PAGES 284–307

If you make mistakes in a row of examples, turn to the page shown beside the row. Study the explanation and repeat the practice. Then try the row on this page again.

Copy. Multiply. Check your work.

284 1. $\$7.54 \times 5$ $\quad \$8.75 \times 9$ $\quad \$34.06 \times 6$ $\quad \$7.64 \times 27$ $\quad \$92.34 \times 34$

284 2. $\$.47 \times 6$ $\quad \$.65 \times 8$ $\quad \$.80 \times 9$ $\quad \$.57 \times 70$ $\quad \$.80 \times 40$

289 3. 6.42×7 $\quad 3.07 \times 9$ $\quad 8.40 \times 26$ $\quad .85 \times 5$ $\quad .06 \times 38$

291 4. 5.214×8 $\quad 3.234 \times 6$ $\quad .826 \times 5$ $\quad .716 \times 29$ $\quad 4.365 \times 34$

292 5. $76 \times .8$ $\quad 180 \times .6$ $\quad 4\,5 \times 3.7$ $\quad 3\,64 \times 1.24$ $\quad 3654 \times .706$

294 6. $.7 \times .5$ $\quad 4.7 \times .6$ $\quad .85 \times 1.7$ $\quad 4.53 \times 7.2$ $\quad 36.1 \times 4.23$

304 7. 10×3.65 $\quad 100 \times 8.47$ $\quad 1000 \times 5.32$

Copy. Divide. Check your work.

298 8. $5\overline{)18.5}$ $\quad 7\overline{)17.64}$ $\quad 8\overline{)66.736}$ $\quad 37\overline{)26.27}$ $\quad 43\overline{)3.612}$

298 9. $2\overline{).6}$ $\quad 6\overline{).36}$ $\quad 4\overline{).16}$ $\quad 9\overline{)82.62}$ $\quad 5\overline{).35}$

300 10. Change the following fractions to decimals. Round to the nearest hundredth.

$\frac{3}{7}$ $\quad \frac{2}{3}$ $\quad \frac{5}{12}$ $\quad \frac{3}{16}$ $\quad \frac{8}{9}$ $\quad \frac{4}{15}$

286 11. Round to the nearest tenth: .38 .246 .472

Copy. Divide. Check your work.

12. $.4\overline{)24}$ $.6\overline{)42}$ $1.5\overline{)120}$ $2.5\overline{)200}$ $4.3\overline{)2537}$

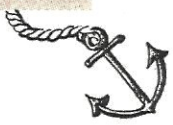
13. $.03\overline{)27}$ $.07\overline{)49}$ $.21\overline{)42}$ $.64\overline{)384}$ $.25\overline{)50}$

14. $.4\overline{)4.8}$ $.42\overline{)5.88}$ $.06\overline{).36}$ $2.4\overline{)55.2}$ $6.23\overline{)8.3482}$

307

307

307



Problems to Solve

1. Sue buys a 12-ride railroad ticket for $2.40. How much does each ride cost?
2. The floor area of the dining section of a railroad dining car is 348.75 square feet. The car holds 36 persons. To the nearest tenth square foot, how much floor area is allowed per person?
3. Bob wants to know what the average monthly rainfall is in the city where he lives. He found that the average yearly rainfall is 42.60 inches. What is the average monthly rainfall?
4. In 1911 a famous speed skater skated 220 yards in 18 seconds. To the nearest tenth yard, what was the average distance he skated in one second?
5. In 1948 a famous runner ran 100 yards in 9.3 seconds. To the nearest tenth, what was his average speed in yards per second?
6. The distance by train from Washington, D. C. to Boston, Mass. is 456.1 miles. A train which leaves Washington at 9:45 A.M. arrives in Boston at 6:45 P.M. How many hours does the trip take? To the nearest tenth mile, what is the average distance the train travels in an hour?
7. The air distance from New York to Paris is 3600 miles. To the nearest whole number, what is the average speed of an airliner that makes the trip in 11.5 hours? (This speed should be labeled as *miles per hour*.)
8. The average weight of a carload of tobacco is 12.84 tons. The average weight of a carload of apples is 19.61 tons. Would you say that apples are about $1\frac{1}{2}$ times as heavy as tobacco? Find the exact figure in decimal form correct to the nearest hundredth.

What Have You Learned about Multiplying and Dividing Decimals?

CHAPTER TEST ON MEANINGS AND SKILLS

1. How many .1's are there in 1? How many .01's in 1?
2. How many .01's are there in .1?
3. Round each of the following to the nearest hundredth.

   4.638 2.473 7.413 18.845 17.375 36.077

4. Change the following fractions to decimals. Express the repeating decimals as hundredths with a fraction; for example, $\frac{5}{6} = .83\frac{1}{3}$.

   $\frac{1}{2}$ $\frac{1}{5}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{5}{8}$ $\frac{1}{6}$ $\frac{7}{8}$ $\frac{1}{7}$ $\frac{4}{5}$ $\frac{5}{20}$

5. Divide the following and round the quotient to the nearest tenth. Carry the work to two decimal places.

   $24\overline{)5.61}$ $37\overline{)162.83}$ $45\overline{)32}$ $16\overline{)62}$ $34\overline{)73.7}$

Copy. Multiply. Check your work.

6. 3.7 × 5 | 2.76 × 8 | 3.785 × 9 | 22.6 × 24 | 3.016 × 57

7. .3 × .2 | .04 × .2 | .009 × .7 | 2.36 × .07 | 1.04 × 43.7

8. 2.34 × .17 | .67 × .84 | 6.09 × 4.76 | 2.671 × 50 | 3.69 × .008

Copy. Divide. Check your work.

9. $2\overline{).4}$ $3\overline{)3.6}$ $5\overline{)2.75}$ $71\overline{)24.85}$ $82\overline{)68.88}$

10. $.4\overline{)8}$ $.5\overline{)35}$ $.8\overline{)248}$ $.21\overline{)882}$ $.07\overline{)693}$

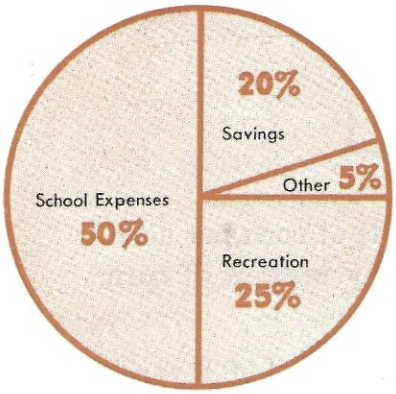
11. $.08\overline{)48}$ $.2\overline{).8}$ $.07\overline{)4.2}$ $3.5\overline{)7.35}$ $.72\overline{).864}$

12. $.007\overline{).063}$ $.42\overline{)77.7}$ $.06\overline{)43.2}$ $2.7\overline{)14.31}$ $1.44\overline{)68.4}$

What Have You Learned about Multiplying and Dividing Decimals?

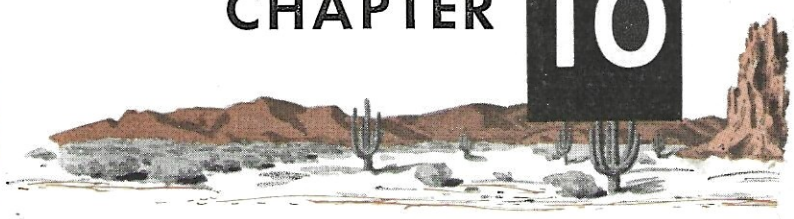
CHAPTER TEST ON PROBLEM SOLVING

1. Mr. Morris raised an average of 32.4 bushels of corn per acre. He had 23 acres planted in corn. How many bushels of corn did he raise?
2. One of Mr. Morris's fields was 381.6 yards long and 98.8 yards wide. What was the area of the field in square yards?
3. Mrs. Jackson bought 7.5 pounds of meat for $4.65. What was the price of one pound of this meat?
4. From 18.9 pounds of carrots Mr. Nolan made 21 bunches. What was the average weight of a bunch?
5. A gallon of milk weighs 8.6 pounds. How many pounds of milk are there in 15.5 gallons?
6. The present size of a dollar bill in the United States is 2.61 inches by 6.14 inches. How many square inches of paper are used for one bill?
7. Mr. Morris's chickens ate 93.4 pounds of feed in one week. At this rate, how many pounds did they eat in 4.5 weeks?
8. A carton of 24 cans of fruit weighs 38.4 pounds. What is the weight of one can of this fruit?
9. Mrs. Morris sold 24.5 pounds of dressed chickens at 63 cents a pound. How much did she receive for the chickens?
10. The school baseball team played 14 games and won 8 of them. What part of the games did they win? Express your answer as a decimal. Round to thousandths.
11. Jim's favorite baseball team played 125 games and won .584 of them. How many games did they win?
12. Sally Morris helped gather 12 dozen eggs for the market. The average weight of a dozen was 24.2 ounces. What was the weight of all these eggs?



How Ann Spends Her Allowance

CHAPTER 10



Per Cent

MEANING; FINDING A PER CENT OF A NUMBER

Ann drew a circle graph to show how she usually spends her allowance. She does not buy her clothes with her allowance, but she uses it for school lunches and all other school expenses, movies and other recreations, savings, and various other things.

Notice that Ann used *per cents* (%) in making her graph.

The words *per cent* mean *per hundred*, or *hundredths*. The sign % is usually used in place of the words *per cent*. Now you know three ways to write any number of hundredths. For example, $\frac{13}{100} = .13 = 13\%$. Also, since *per cent* means *hundredths*, it is very easy to express any per cent as a decimal or a common fraction. For example, $25\% = .25 = \frac{25}{100}$. The common fraction can then be changed to lowest terms, if you want to do that.

1. Copy in a column each of the per cents which Ann used in her graph. Opposite each per cent write the decimal fraction and the common fraction which have the same value. Change each common fraction to lowest terms.

2. Can you tell from the graph that $50\% = \frac{1}{2}$? That $25\% = \frac{1}{4}$?

Per cents of the same quantity may be added just as any like numbers are added.

3. Add $50\% + 25\% + 20\% + 5\%$ of Ann's allowance. Does your sum represent all of Ann's allowance? Is that 100% of it? Is *the whole* of anything 100% of it?

Building upon What You Know

4. What fraction of a dollar is 1 cent? What per cent of a dollar is 1 cent? 10 cents? 25 cents? 50 cents? 87 cents? 100 cents? $33\frac{1}{3}$ cents? $12\frac{1}{2}$ cents?
5. Would it be correct to say that Ann spends 50 cents out of every dollar (or every 100 cents) for school expenses? How many cents out of every dollar does she spend for recreation? How many cents out of every dollar does she save?
6. Ann's allowance is $3.00 per week. Find the amount that Ann spends each week for school expenses by using the decimal that means the same as 50%. Check your answer by using the fraction which equals 50%.
7. In the same two ways suggested in exercise 6, find the amounts that Ann uses for recreation, for savings, and for other things.

By changing each per cent to a decimal fraction, find the following products. Check by changing the per cent to a common fraction in lowest terms and multiplying again.

8. 50% of 18 gallons — 25% of 2084 books
9. 20% of 475 pupils — 75% of 40 answers
10. 60% of $75 — 80% of 500 miles

Solve the following problems.

11. John had 20 questions to answer for homework. He did 50% of them before supper. How many questions did he do before supper?
12. Jane made a score of 92% on a social studies test. There were 25 questions on the test. How many questions did she answer correctly?

Three Kinds of Problems Using Per Cents

All of the problems on this page fit the model sentence "___% of ___ = ___." Read each problem carefully. Decide what a sensible answer might be. Write the problem in the form of the model sentence, find the missing number, and check your answer.

1. 20% of 80 is what number?
2. What per cent of 500 is 10?
3. 40% of a number is 7. What is the number?
4. My father says that I may have 30% of the pennies in our "piggy bank." There are 160 pennies in the bank. How many may I have?
5. Phil sold his clarinet for $50. He spent 75% of this for a guitar. How much did he spend for the guitar?
6. The Old Home Bakery made 180 cakes. They sold 171 of them. What per cent of the cakes did they sell?
7. Joe worked for 45 minutes mowing his lawn. What per cent of an hour did he work?
8. Arthur says that 90% of all his chickens are pullets. He has 180 pullets. How many chickens in all does he have?
9. Near the end of the year, we bought a new car for $3200. The dealer said that this was 75% of the original price. What was the original price?
10. There are 680 pupils in Cedar School. 45% of them went on the school trip. How many went on the school trip?
11. On a very stormy day, 121 of the 680 pupils were absent. To the nearest whole per cent, what per cent were absent?
12. When the gasoline gauge on our car first registers empty, there are actually 2.7 gallons of gasoline in the tank. This is 15% of the tank's capacity. How much does the tank hold?

New Thoughts about Whole Numbers

ENRICHMENT UNIT: PRIME FACTORS, EXPONENTS, POWERS

Read the following paragraphs slowly and thoughtfully, one sentence at a time. Follow directions carefully. It is understood that whenever we say *number*, we mean *whole number*.

On a piece of paper, write the numbers from 2 to 100, spaced so that you can read them easily. Circle the 2, then cross out every other number which is 2 times some number. These numbers are *multiples* of 2. Have you crossed out every even number except 2?

Circle the 3, then cross out the larger numbers which are multiples of 3. You have already crossed out the first one, 6. Why is 6 a multiple of both 2 and 3? Cross out 9, and go on from there.

The next number is 4. It is already crossed out. Is every multiple of 4 also a multiple of 2? Then every multiple of 4 is crossed out.

Circle the 5, then cross out every larger number which is a multiple of 5. Is 10 a multiple of both 2 and 5? What do you expect to find as you cross out the multiples of 5?

Six is already crossed out. Is every multiple of 6 also a multiple of 2 and 3?

Circle the 7, then cross out the multiples of 7.

The next number not crossed out is 11. Are all the larger multiples of 11 crossed out? If you extended your list of numbers beyond 100, would you expect to find multiples of 11 to cross out? ($11 \times 11 =$ _?_) The same ideas apply to the other numbers which are not crossed out in your list of numbers to 100. Circle them.

The numbers you have circled are called *prime numbers*. Prime numbers have no factors other than themselves and 1. Another way to say this is that if you divide a prime number by any number except itself and 1, you will get a remainder. We shall agree not to call 1 a prime number.

1. Refer to your paper. List the prime numbers smaller than 100. How many of them are there? Do you think there would be any

prime numbers between 100 and 200? Do you think there might be prime numbers larger than 500? larger than 1,000,000?

2. There is only one prime number which is an even number. What is it? Why are there no more even primes?

3. Write a pair of factors of 15. Do not use 15×1. Are both of your factors prime numbers?

4. Write a pair of factors for each of the following numbers. Circle each factor which is a prime number.
   6 8 10 14 20 36 50 49 11

5. Write a pair of factors of 12. Do not use 12×1. Are both of these factors prime numbers? Write the factors of the number which is not prime. Now write 12 as the product of three factors. Are all of these three factors prime numbers?

You have found the *prime factors* of 12. There are two ways you might have done this, not counting the different orders of the first pair of factors:

a) $12 = 3 \times 4$
   but $4 = 2 \times 2$
   so $12 = 3 \times 2 \times 2$

b) $12 = 2 \times 6$
   but $6 = 2 \times 3$
   so $12 = 2 \times 2 \times 3$

The commutative and associative laws guarantee that the order in which several factors are written does not affect the product. You may conclude that, no matter how you start in factoring 12, if you make no mistakes, you always get the same set of prime factors.

6. Find the prime factors of 30, first starting with 5×6, then 10×3. Did you get the same set of prime factors each time?

7. Find the prime factors of 36. Use 2×18, then 4×9, then 6×6. Did you get the same set of prime factors each time?

8. Find the prime factors of each of the following numbers.
   18 49 28 32 56 30 81 100 98

Let us see how to find factors of numbers whose factors we cannot tell at a glance, 405, for instance. Every number whose numeral ends in 5 is a multiple of 5. Divide 405 by 5. Did you find that $405 = 5 \times 81$? Can you factor the 81? What are the prime factors of 405?

When factors are repeated, as in $3 \times 3 \times 3 \times 3 \times 5$, we have a short way to write the multiplication. For $3 \times 3 \times 3 \times 3$, we can write 3^4. The little 4 written at the upper right-hand corner of the 3 is used as a factor 4 times. In this example the 4 is called an *exponent*. The 3 is called the *base*. Since $81 = 3^4$, 81 is said to be the fourth *power* of 3. You may read $81 = 3^4$ as "81 equals 3 to the fourth power." Is it true that $405 = 5 \times 3^4$?

The second and third powers of a number may be read in another way. What are the factors of 25? $5 \times 5 = 5^2$. $25 = 5^2$ may be read as "25 equals 5 to the second power" or "25 equals 5 *squared*." What are the prime factors of 8? $2 \times 2 \times 2 = 2^3$. $8 = 2^3$ may be read as "8 = 2 to the third power" or "8 = 2 *cubed*."

9. Use exponents in writing each of the following numbers as the product of prime factors.
   4 9 16 27 125 49 343 625 64

10. Show that $24 = 3 \times 2^3$, or $72 = 3^2 \times 2^3$.

11. Write each of the following numbers in the form used in exercise 10.
    12 36 20 56 54 40 50 75 100

The exponential form may be used with numbers that are not prime numbers. For instance, since $100 = 10 \times 10 = 10^2$, we can say that 100 is a power of 10.

12. Show that 1,000 and 10,000 are both powers of 10. What do you notice about the exponent and the number of zeros it takes to write the numerals in these cases?

13. Write each of the following as a power of 10.
    100,000 1,000,000 10,000,000

Since $200 = 2 \times 10^2$, 200 is a multiple of a power of 10.

14. Write each of the following as a multiple of a power of 10.
    300 500 700 2000 5000 20,000 70,000

You know that fractions written in decimal form have "unwritten denominators" of 10, 100, and so forth. In exponential form 10 may be written as 10^1, 100 may be written as 10^2. How can you write 1000 in exponential form? Then you may write $.326 = \frac{326}{1000} = \frac{326}{10^3}$.

15. Tell the "unwritten denominator" of each of the following decimals. Is each of them a power of 10?
    .4 .17 .05 .398 .206 .1976 .0003 .3008

16. Change each of these common fractions into an equivalent fraction whose denominator is a power of 10. For example, $\frac{2}{5} = \frac{4}{10} = \frac{4}{10^1}$; $\frac{3}{4} = \frac{75}{100} = \frac{75}{10^2}$.

    $\frac{1}{2}$ $\frac{1}{4}$ $\frac{4}{5}$ $\frac{7}{25}$ $\frac{1}{8}$ $\frac{3}{8}$ $\frac{1}{16}$ $\frac{5}{16}$

Turn back to page 301 and read the paragraphs about terminating and repeating decimals. If a common fraction is to convert into a terminating decimal ("come out even" when the numerator is divided by the denominator), it must have a denominator which can be changed to a power of 10 by multiplying by some whole number. All of the fractions in exercise 16 are fractions of this sort.

17. Write each of the fractions in exercise 16 in decimal form.

18. Find the prime factors of the denominators of the fractions in exercise 16.

19. In exercise 16, when you changed the fractions so that their denominators would be powers of 10, did you always multiply by either a power of 2 or a power of 5?

To tell whether a common fraction will convert into a terminating decimal or a repeating decimal, find the prime factors of the denominator. If any number except 2 and 5 is one of these prime factors, the fraction will convert into a repeating decimal.

Final Test on the Year's Work

Vocabulary

1. Which of the following is an eight-place number?

   (a) 808 (b) 836,838 (c) 82,364,898 (d) 8,367,428

2. Write the following words in a column. Beside each word write a number from the examples below which shows that you know what the word means.

(a) sum	(e) subtrahend	(i) dividend
(b) addend	(f) multiplier	(j) divisor
(c) difference	(g) multiplicand	(k) quotient
(d) minuend	(h) product	(l) remainder

$$\begin{array}{r} 276 \\ +145 \\ \hline 421 \end{array} \qquad \begin{array}{r} 687 \\ -296 \\ \hline 391 \end{array} \qquad \begin{array}{r} 463 \\ \times 27 \\ \hline 3241 \\ 926 \\ \hline 12501 \end{array} \qquad \begin{array}{r} 41 \text{ r}9 \\ 64\overline{)2633} \\ 256 \\ \hline 73 \\ 64 \\ \hline 9 \end{array}$$

3. In the fraction $\frac{8}{9}$, the numerator is _?_; the denominator is _?_.

4. A proper fraction has a numerator that is _?_ than the denominator.

5. A fraction whose value is 1 or more than 1 is called an _?_ fraction.

6. A mixed number has a _?_ and a _?_.

7. Write $\frac{7}{10}$ as a decimal fraction.

8. In multiplication and division of fractions, cancellation shows that both a numerator and a denominator have been _?_ by the same number.

9. The dimensions of a rectangle are its _?_ and _?_.

10. If the length and width of a rectangle are measured in feet, its area will be measured in _?_ _?_.

Meanings

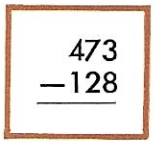
1. In the number 6,784.531, the 6 stands for 6 _?_, the 7 stands for 7 _?_, the 8 stands for 8 _?_, the 4 stands for 4 _?_, the 5 stands for 5 _?_, the 3 stands for 3 _?_, the 1 stands for 1 _?_.

2. Which of the following numbers means 7 thousands 6 hundreds 4 tens 5 ones?

   6754 7456 5467 7645

3. In the example at the left, after adding the ones column, think of the 2 in 23 ones as _?_ tens and add it to the numbers in the _?_ column.

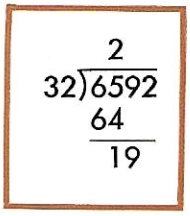
4. Use $27 + 34 + 56$ to show that changing the order of the addends does not change the sum.



5. In the example at the left, when you borrow 1 ten, the 7 tens are changed to _?_ tens and the 3 ones are changed to _?_ ones.

6. Add $312 + 312 + 312 + 312$. Multiply 4×312. Multiplication is a quick way of _?_ equal numbers.

7. When you multiply 45×76, you multiply _?_ ones $\times$ 76 and _?_ tens $\times$ 76.



8. What is the next step in the example at the left?
   (a) Bring down the 2.
   (b) Write 0 in the quotient over 9.
   (c) Divide 32 by 19.
   (d) Multiply 32 by 6.

9. Start with 102 and subtract 34 as many times as you can. Divide 102 by 34. The quotient tells how many times the divisor can be _?_ from the dividend.

10. Which is larger? $\frac{1}{4}$ or $\frac{1}{8}$ $\frac{1}{2}$ or $\frac{1}{3}$ $\frac{1}{9}$ or $\frac{1}{7}$

11. The decimal fraction .18 shows that 18 of _?_ equal parts are spoken of.

Computation

1. Write the number for: one hundred ten million, six hundred seven thousand, two hundred forty-five.
2. Change $\frac{1}{8}$ to a decimal thousandth.
3. Write the Arabic numerals for XIV; CCV; MCMLX.

Copy and do as the signs tell you to do.

4. $\begin{array}{r} 476 \\ 307 \\ 892 \\ +354 \end{array}$ $\begin{array}{r} 8504 \\ 2967 \\ 6790 \\ +4865 \end{array}$ $\begin{array}{r} 9\frac{1}{9} \\ +3\frac{2}{9} \end{array}$ $\begin{array}{r} 12\frac{1}{3} \\ +4\frac{3}{4} \end{array}$ $\begin{array}{r} 15.017 \\ 27.268 \\ 4.596 \\ +18.260 \end{array}$

5. $\begin{array}{r} 8657 \\ -2945 \end{array}$ $\begin{array}{r} 23.75 \\ -15.86 \end{array}$ $\begin{array}{r} 7\frac{5}{6} \\ -2\frac{1}{6} \end{array}$ $\begin{array}{r} 15\frac{1}{6} \\ -10\frac{2}{3} \end{array}$ $\begin{array}{r} 9\frac{1}{3} \\ -3\frac{1}{2} \end{array}$

6. $\begin{array}{r} 365 \\ \times 34 \end{array}$ $\begin{array}{r} 506 \\ \times 56 \end{array}$ $\begin{array}{r} 76.3 \\ \times 72 \end{array}$ $\begin{array}{r} 800 \\ \times 6.27 \end{array}$ $\begin{array}{r} 3.81 \\ \times 5.6 \end{array}$

7. $53\overline{)389}$ $42\overline{)896}$ $5.3\overline{)15.75}$ $.5\overline{)8.753}$ $.37\overline{)8.586}$

8. Find the perimeters of rectangles with the following dimensions: 36 ft. by 24 ft.; 18 in. by 32 in.
9. Find the areas of rectangles with the following dimensions: 15 ft. by 19 ft.; 26 yd. by 32 yd.; 27 in. by 36 in.
10. 432 sq. in. = _?_ sq. ft.; 90 sq. ft. = _?_ sq. yd.; 8 sq. ft. = _?_ sq. in.
11. 60 in. = _?_ ft.; 9 gal. = _?_ qt.; 64 oz. = _?_ lb.

Do as the signs tell you to do.

12. $\frac{1}{7}+\frac{3}{7}$ $\frac{3}{4}+\frac{3}{8}$ $7\frac{1}{3}-3\frac{5}{6}$ $12\frac{7}{12}-7\frac{3}{4}$ $8\frac{1}{5}-3\frac{1}{2}$
13. $\frac{3}{8}\times 24$ $\frac{3}{7}\times 4\frac{2}{3}$ $5\frac{2}{5}\times 35$ $\frac{5}{8}\times\frac{16}{25}$ $2\frac{1}{4}\times 3\frac{1}{2}$
14. $6\div\frac{3}{7}$ $\frac{1}{2}\div\frac{5}{8}$ $7\frac{1}{2}\div\frac{3}{8}$ $8\frac{3}{4}\div 5$ $4\frac{1}{5}\div 3\frac{1}{2}$

Problems

1. What was the average speed of a plane that flew 185 miles the first hour, 205 miles the second, and 195 miles the third?
2. Mr. Smith drove 308.5 miles on Monday and 279.8 miles on Tuesday. How many more miles did he drive on Monday than on Tuesday?
3. Janice paid 59 cents a yard for 5 yards of gingham. She gave the clerk $5.00. How much change did she receive?
4. How many square inches of linoleum are needed to cover a table top that is 52 inches by 29 inches? What is the perimeter of the table top?
5. In a large city one newspaper has a daily sale of 260,692 copies and another newspaper has a daily sale of 173,865 copies. What is the difference in the daily sales of these newspapers?
6. Jack caught two fish. One weighed $2\frac{3}{4}$ pounds and the other $3\frac{1}{2}$ pounds. How much did both weigh?
7. Mrs. Jones roasted $5\frac{1}{2}$ pounds of beef. The family ate $2\frac{3}{4}$ pounds at the first meal. How many pounds were left?
8. John walked the first mile in 15 minutes 25 seconds and the second mile in 17 minutes 52 seconds. How long did it take him to walk both miles?
9. A train left Washington, D. C. at 10:00 A.M. and arrived in New York City at 2:00 P.M., an approximate distance of 224 miles. What is the average speed of the train?
10. Chickens are on sale at 48 cents a pound. How much will a chicken weighing $4\frac{3}{4}$ pounds cost?
11. A card is $3\frac{1}{4}$ inches by $5\frac{1}{2}$ inches. What is the area of the card?
12. Dorothy is making a poster for the school fair. Her lines for lettering are 15 inches long. If she allows $1\frac{1}{2}$ inches for each letter, how many letters can she print on a line?

100 Addition Facts

0 0 0	0 1 1	0 2 2	0 3 3	0 4 4	0 5 5	0 6 6	0 7 7	0 8 8	0 9 9
1 0 1	1 1 2	1 2 3	1 3 4	1 4 5	1 5 6	1 6 7	1 7 8	1 8 9	1 9 10
2 0 2	2 1 3	2 2 4	2 3 5	2 4 6	2 5 7	2 6 8	2 7 9	2 8 10	2 9 11
3 0 3	3 1 4	3 2 5	3 3 6	3 4 7	3 5 8	3 6 9	3 7 10	3 8 11	3 9 12
4 0 4	4 1 5	4 2 6	4 3 7	4 4 8	4 5 9	4 6 10	4 7 11	4 8 12	4 9 13
5 0 5	5 1 6	5 2 7	5 3 8	5 4 9	5 5 10	5 6 11	5 7 12	5 8 13	5 9 14
6 0 6	6 1 7	6 2 8	6 3 9	6 4 10	6 5 11	6 6 12	6 7 13	6 8 14	6 9 15
7 0 7	7 1 8	7 2 9	7 3 10	7 4 11	7 5 12	7 6 13	7 7 14	7 8 15	7 9 16
8 0 8	8 1 9	8 2 10	8 3 11	8 4 12	8 5 13	8 6 14	8 7 15	8 8 16	8 9 17
9 0 9	9 1 10	9 2 11	9 3 12	9 4 13	9 5 14	9 6 15	9 7 16	9 8 17	9 9 18

100 Subtraction Facts

0 0 0	1 0 1	2 0 2	3 0 3	4 0 4	5 0 5	6 0 6	7 0 7	8 0 8	9 0 9
1 1 0	2 1 1	3 1 2	4 1 3	5 1 4	6 1 5	7 1 6	8 1 7	9 1 8	10 1 9
2 2 0	3 2 1	4 2 2	5 2 3	6 2 4	7 2 5	8 2 6	9 2 7	10 2 8	11 2 9
3 3 0	4 3 1	5 3 2	6 3 3	7 3 4	8 3 5	9 3 6	10 3 7	11 3 8	12 3 9
4 4 0	5 4 1	6 4 2	7 4 3	8 4 4	9 4 5	10 4 6	11 4 7	12 4 8	13 4 9
5 5 0	6 5 1	7 5 2	8 5 3	9 5 4	10 5 5	11 5 6	12 5 7	13 5 8	14 5 9
6 6 0	7 6 1	8 6 2	9 6 3	10 6 4	11 6 5	12 6 6	13 6 7	14 6 8	15 6 9
7 7 0	8 7 1	9 7 2	10 7 3	11 7 4	12 7 5	13 7 6	14 7 7	15 7 8	16 7 9
8 8 0	9 8 1	10 8 2	11 8 3	12 8 4	13 8 5	14 8 6	15 8 7	16 8 8	17 8 9
9 9 0	10 9 1	11 9 2	12 9 3	13 9 4	14 9 5	15 9 6	16 9 7	17 9 8	18 9 9

100 Multiplication Facts

0 × 0 = 0	1 × 0 = 0	2 × 0 = 0	3 × 0 = 0	4 × 0 = 0	5 × 0 = 0	6 × 0 = 0	7 × 0 = 0	8 × 0 = 0	9 × 0 = 0
0 × 1 = 0	1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4	5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8	9 × 1 = 9
0 × 2 = 0	1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8	5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16	9 × 2 = 18
0 × 3 = 0	1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12	5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24	9 × 3 = 27
0 × 4 = 0	1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16	5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32	9 × 4 = 36
0 × 5 = 0	1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20	5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40	9 × 5 = 45
0 × 6 = 0	1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24	5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48	9 × 6 = 54
0 × 7 = 0	1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28	5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56	9 × 7 = 63
0 × 8 = 0	1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32	5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64	9 × 8 = 72
0 × 9 = 0	1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36	5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72	9 × 9 = 81

90 Division Facts

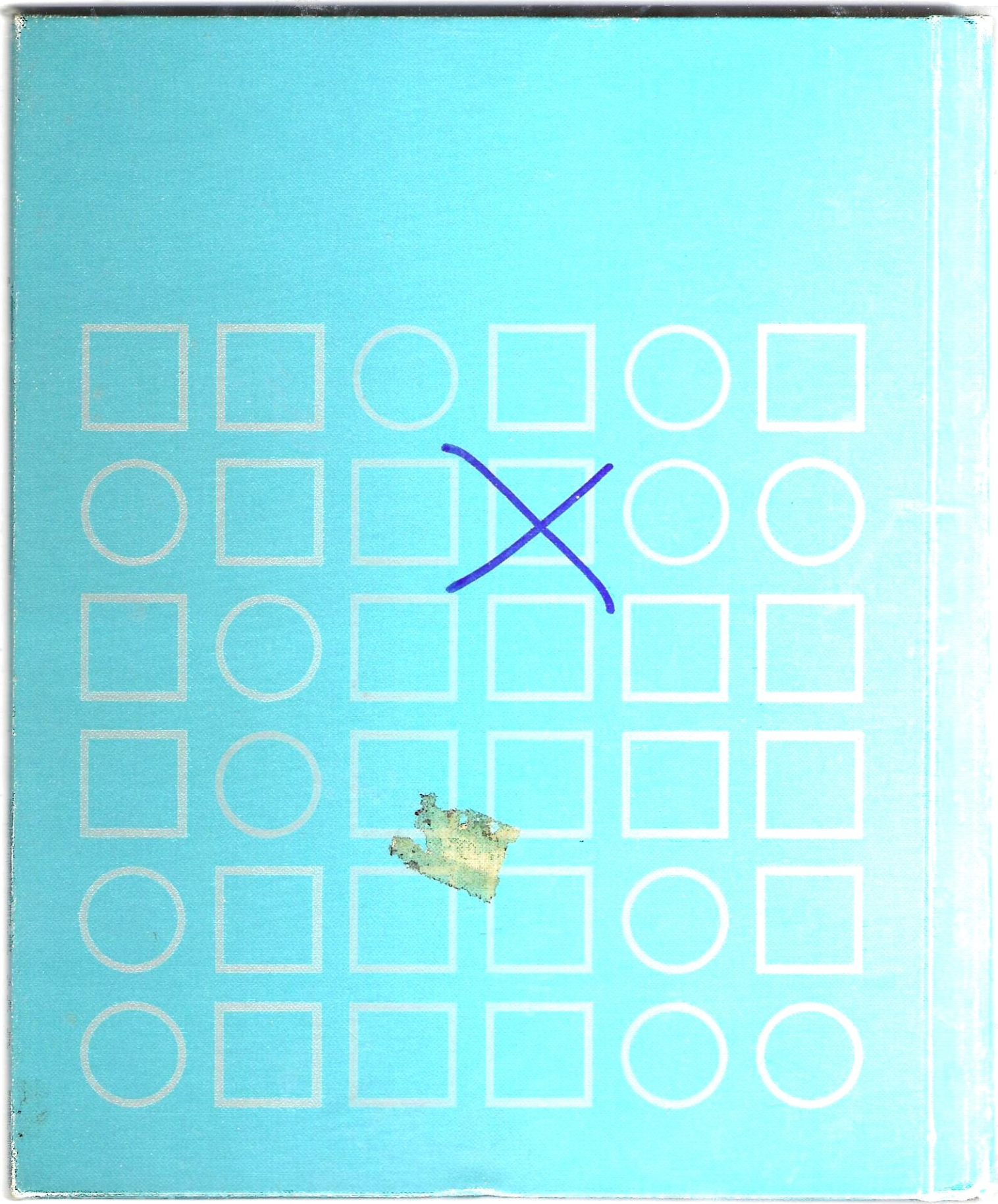
0 ÷ 1 = 0	1 ÷ 1 = 1	2 ÷ 1 = 2	3 ÷ 1 = 3	4 ÷ 1 = 4	5 ÷ 1 = 5	6 ÷ 1 = 6	7 ÷ 1 = 7	8 ÷ 1 = 8	9 ÷ 1 = 9
0 ÷ 2 = 0	1 ÷ 2 = 2	2 ÷ 2 = 4	3 ÷ 2 = 6	4 ÷ 2 = 8	5 ÷ 2 = 10	6 ÷ 2 = 12	7 ÷ 2 = 14	8 ÷ 2 = 16	9 ÷ 2 = 18
0 ÷ 3 = 0	1 ÷ 3 = 3	2 ÷ 3 = 6	3 ÷ 3 = 9	4 ÷ 3 = 12	5 ÷ 3 = 15	6 ÷ 3 = 18	7 ÷ 3 = 21	8 ÷ 3 = 24	9 ÷ 3 = 27
0 ÷ 4 = 0	1 ÷ 4 = 4	2 ÷ 4 = 8	3 ÷ 4 = 12	4 ÷ 4 = 16	5 ÷ 4 = 20	6 ÷ 4 = 24	7 ÷ 4 = 28	8 ÷ 4 = 32	9 ÷ 4 = 36
0 ÷ 5 = 0	1 ÷ 5 = 5	2 ÷ 5 = 10	3 ÷ 5 = 15	4 ÷ 5 = 20	5 ÷ 5 = 25	6 ÷ 5 = 30	7 ÷ 5 = 35	8 ÷ 5 = 40	9 ÷ 5 = 45
0 ÷ 6 = 0	1 ÷ 6 = 6	2 ÷ 6 = 12	3 ÷ 6 = 18	4 ÷ 6 = 24	5 ÷ 6 = 30	6 ÷ 6 = 36	7 ÷ 6 = 42	8 ÷ 6 = 48	9 ÷ 6 = 54
0 ÷ 7 = 0	1 ÷ 7 = 7	2 ÷ 7 = 14	3 ÷ 7 = 21	4 ÷ 7 = 28	5 ÷ 7 = 35	6 ÷ 7 = 42	7 ÷ 7 = 49	8 ÷ 7 = 56	9 ÷ 7 = 63
0 ÷ 8 = 0	1 ÷ 8 = 8	2 ÷ 8 = 16	3 ÷ 8 = 24	4 ÷ 8 = 32	5 ÷ 8 = 40	6 ÷ 8 = 48	7 ÷ 8 = 56	8 ÷ 8 = 64	9 ÷ 8 = 72
0 ÷ 9 = 0	1 ÷ 9 = 9	2 ÷ 9 = 18	3 ÷ 9 = 27	4 ÷ 9 = 36	5 ÷ 9 = 45	6 ÷ 9 = 54	7 ÷ 9 = 63	8 ÷ 9 = 72	9 ÷ 9 = 81

Measures of Length

12 inches = 1 foot
3 feet = 1 yard
36 inches = 1 yard

Liquid Measures

2 cups = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Measures of Time

60 seconds = 1 minute
60 minutes = 1 hour
24 hours = 1 day
7 days = 1 week
365 days = 1 year
366 days = 1 leap year